Lecture-4

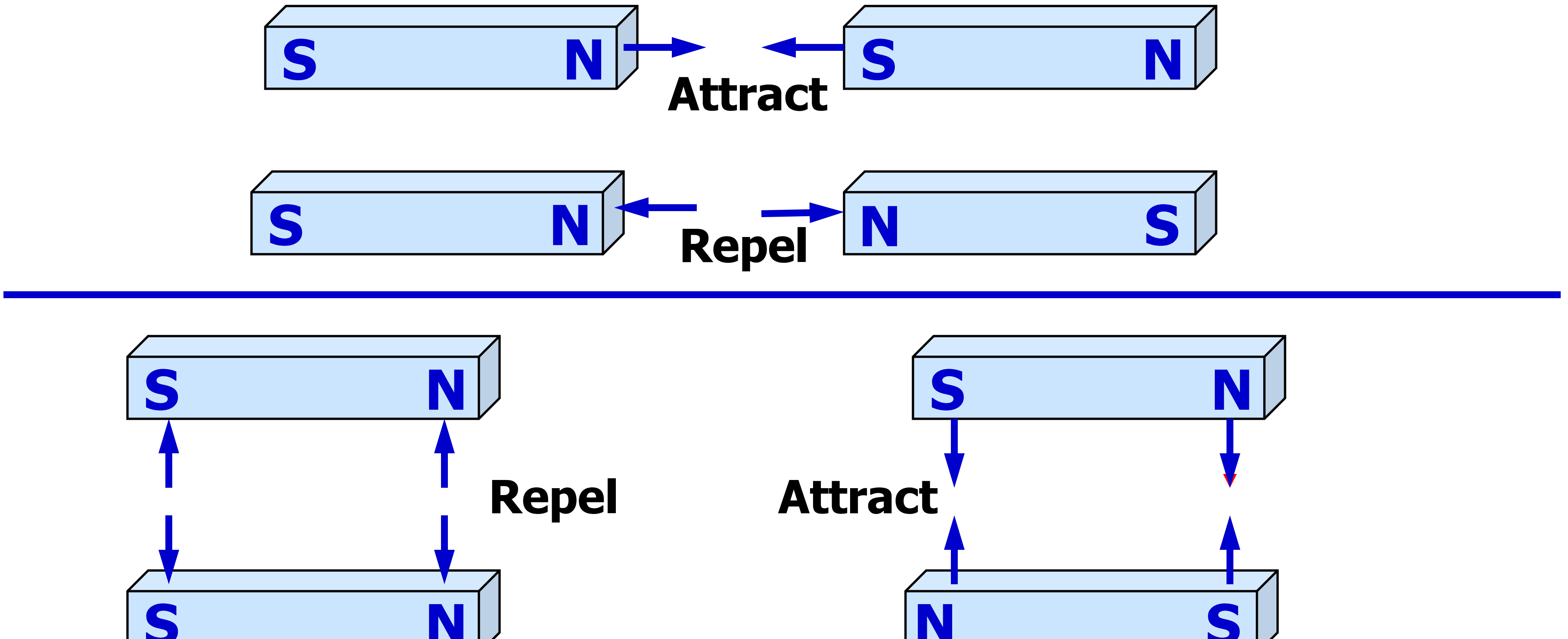
Magnetic forces, Forces due to magnetic field, magnetic torque and moment, a magnetic dipole,

Pradeep Singla

Magnetism

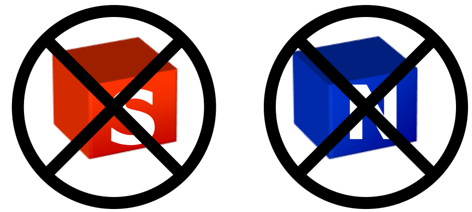
Recall how there are two kinds of electrical charge (+ and -), and likes repel, opposites attract.

Similarly, there are two kinds of magnetic poles (north and south), and like poles repel, opposites attract.

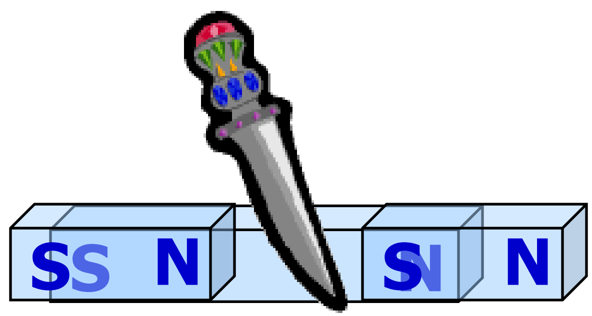


Pradeep Singla

There is one difference between magnetism and electricity: it is possible to have isolated + or – electric charges, but isolated N and S poles have never been observed.



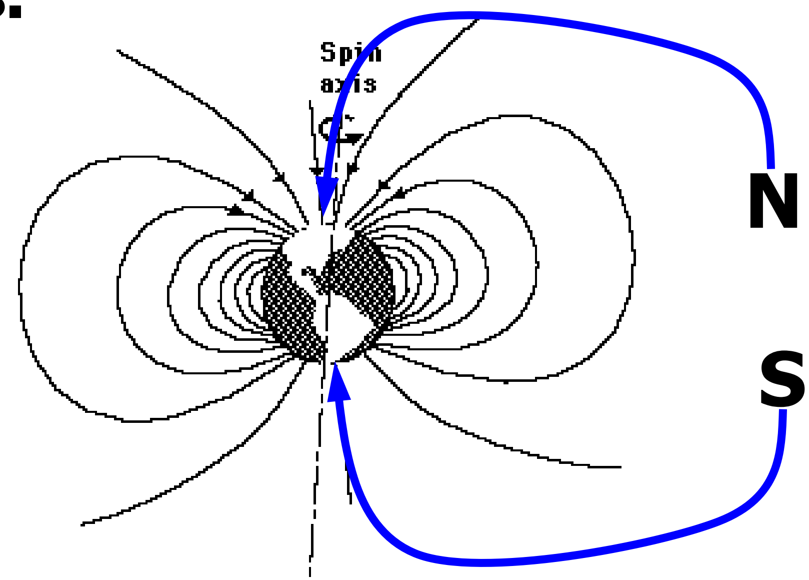
I.E., every magnet has BOTH a N and a S pole, no matter how many times you “chop it up.”



Magnetic Fields

The earth has associated with it a magnetic field, with poles near the geographic poles.

The pole of a magnet attracted to the earth's north geographic pole is the magnet's north pole.
The pole of a magnet attracted to the earth's south geographic pole is the magnet's south pole.



http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/magearth.html

Just as we used the electric field to help us "explain" and visualize electric forces in space, we use the magnetic field to help us "explain" and visualize magnetic forces in space.

Magnetic field lines point in the same direction that the north pole of a compass would point.

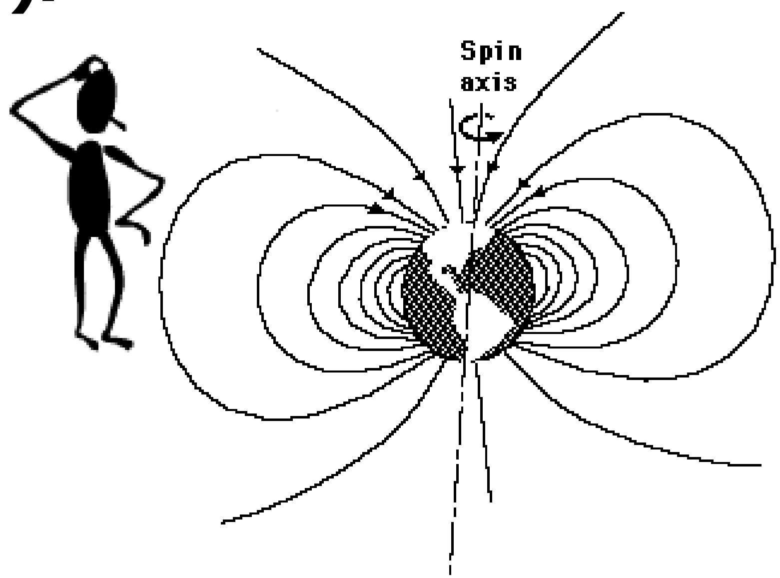
Magnetic field lines are tangent to the magnetic field.

The more magnetic field lines in a region in space, the stronger the magnetic field.

Outside the magnet, magnetic field lines point away from N poles (*why?).

Huh?

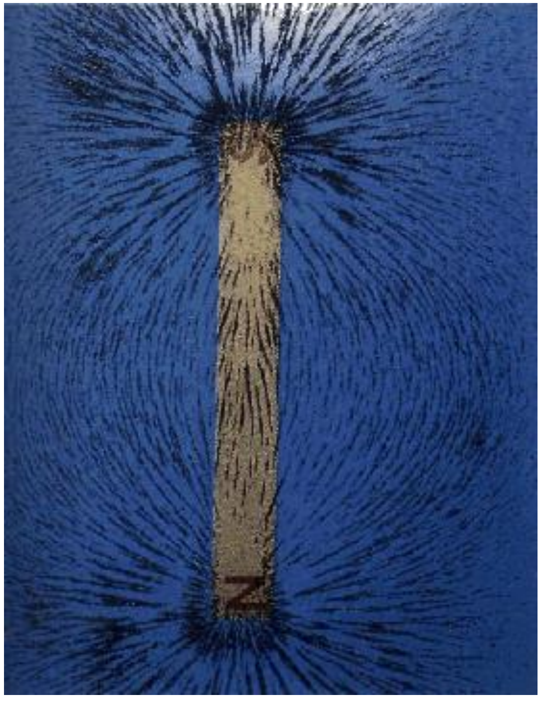
Nooooooo…

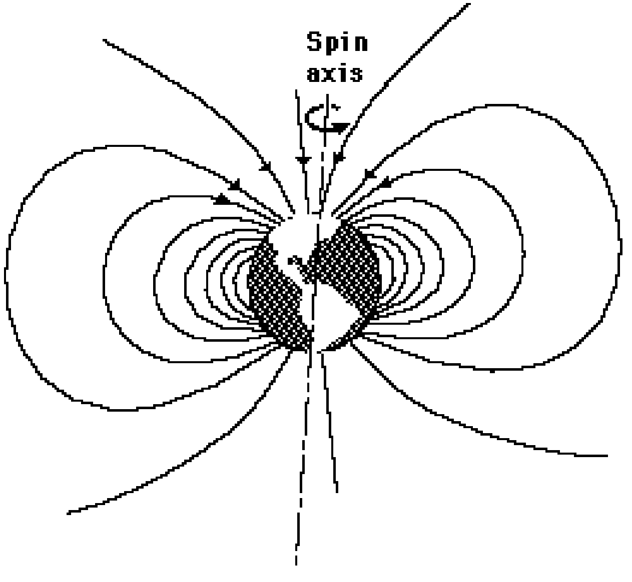


*The N pole of a compass would "want to get to" the S pole of the magnet.

Here's a "picture" of the magnetic field of a bar magnet, using iron filings to map out the field.

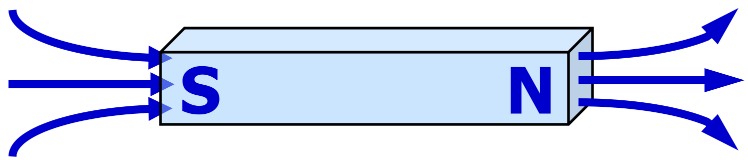
The magnetic field ought to "remind" you of the earth's field.





We use the symbol $\vec{B}$ for magnetic field.

Magnetic field lines point away from north poles, and towards south poles.



The SI unit* for magnetic field is the Tesla.

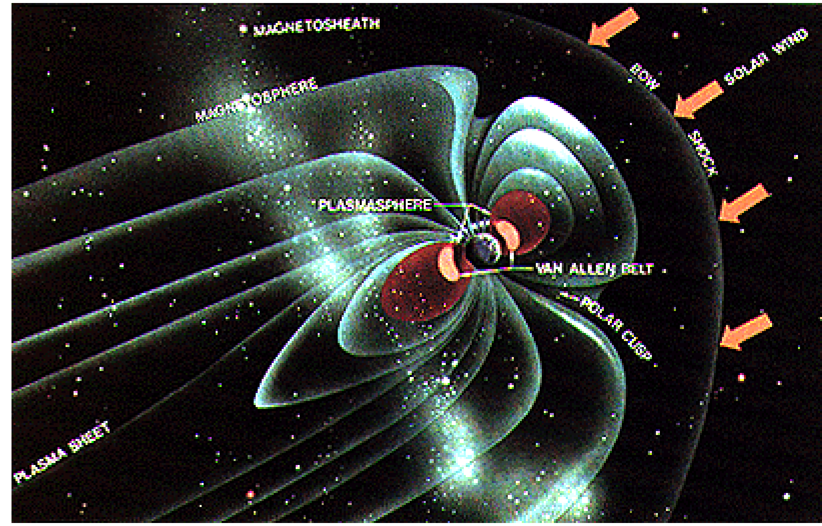
$$1\ \mathrm{T} = \frac{1\ \mathrm{kg}}{\mathrm{C} \cdot \mathrm{s}}$$

These units come from the magnetic force equation, which appears two slides from now.

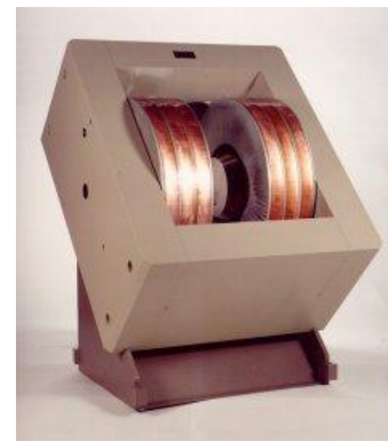
In a bit, we'll see how the units are related to other quantities we know about, and later in the course we'll see an "official" definition of the units for the magnetic field.

***Old unit, still sometimes used: 1 Gauss = 10^{-4} Tesla.**

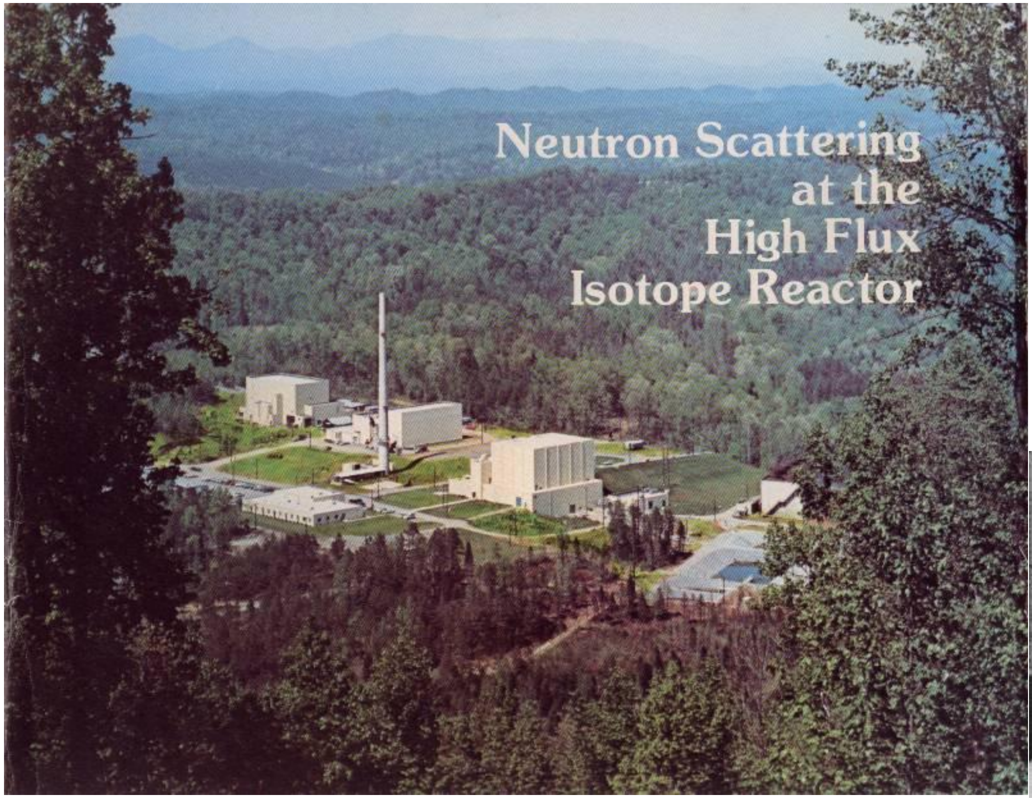
The earth's magnetic field has a magnitude of roughly 0.5 G, or 0.00005 T. A powerful perm-anent magnet, like the kind you might find in headphones, might produce a magnetic field of 1000 G, or 0.1 T.



The electromagnet in the basement of Physics that my students use in experiments can produce a field of 26000 G = 26 kG = 2.6 T.



Superconducting magnets can produce a field of over 10 T. Never get near an operating super-conducting magnet while wearing a watch or belt buckle with iron in it!



Pradeep Singla

Magnetic Fields and Moving Charges

A charged particle moving in a magnetic field experiences a force.

The magnetic force equation predicts the effect of a magnetic field on a moving charged particle.

$$\vec{F} = q\vec{v} \times \vec{B}$$

force on particle — **velocity of charged particle** — **magnetic field vector**

What is the force if the charged particle is at rest?

Vector notation conventions:

⊙

⊙ is a vector pointing out of the paper/board/screen (looks like an arrow coming straight for your eye).

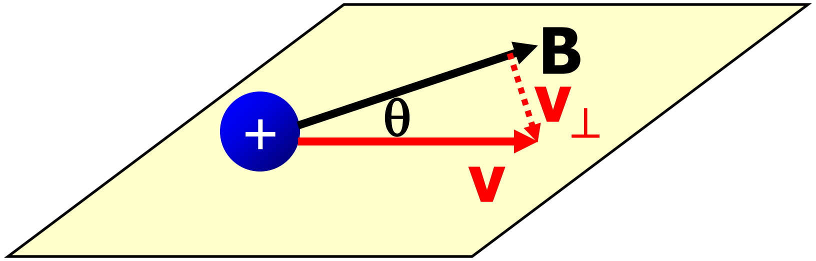
⊗

⊗ is a vector pointing into the paper/board/screen (looks like the feathers of an arrow going away from eye).

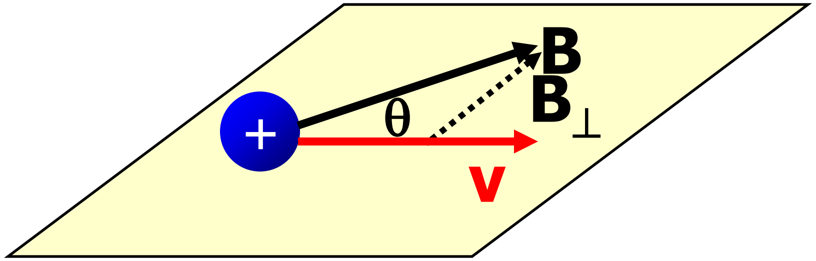
Cross product as presented in Physics 23.

$$F = |q| vB \sin\theta = |q| v_\perp B = |q| vB_\perp$$

Magnitude of magnetic force (and the meaning of $v_\perp$ and $B_\perp$):



$$F = |q| v_\perp B$$



$$F = |q| vB_\perp$$

Direction of magnetic force---

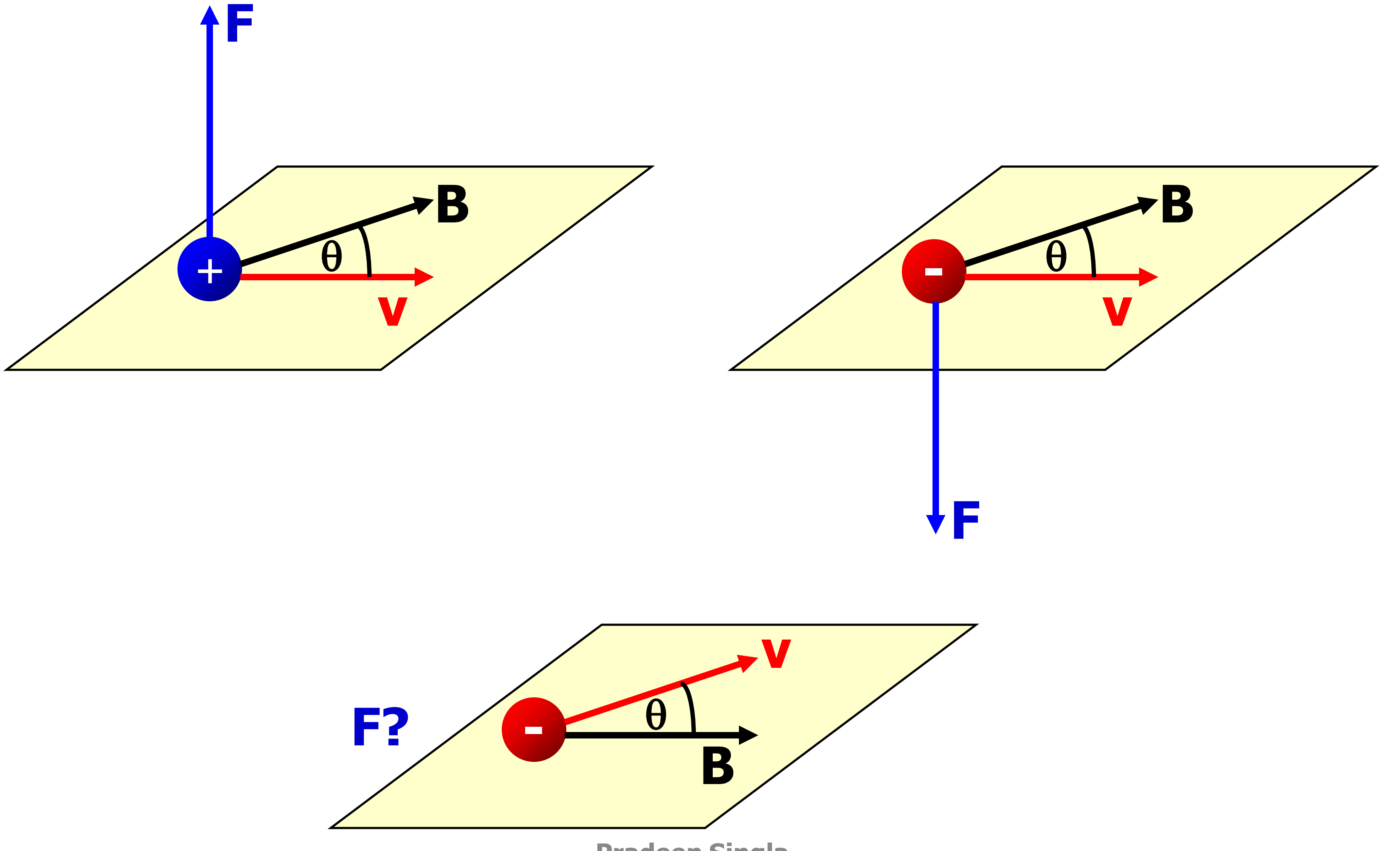
Use right hand rule:

- **fingers out, thumb perpendicular to them**
- **rotate your hand until your palm points in the direction of $\vec{B}$**
- **(or bend fingers through smallest angle from $\vec{v}$ to $\vec{B}$)**
- **thumb points in direction of $\vec{F}$**

Your text presents two alternative variations (curl your fingers, imagine turning a right-handed screw). There is one other variation on the right hand rule. I'll demonstrate all variations in lecture sooner or later.

$$F = |q| vB \sin\theta = |q| v_{\perp} B = |q| v B_{\perp}$$

Direction of magnetic force:



Pradeep Singla

"Foolproof" technique for calculating both magnitude and direction of magnetic force.

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{F} = q \left[\det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{pmatrix} \right]$$

Alternative* view of magnetic field units.

$$F = |q| vB \sin\theta$$

$$B = \frac{F}{|q| v \sin\theta}$$

$$[B] = T = \frac{N}{C \cdot m/s} = \frac{N}{A \cdot m}$$

Remember, units of field are force per "something."

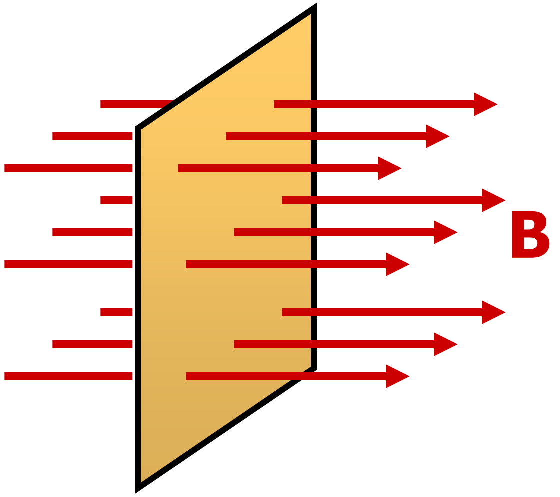
*"Official" definition of units coming later.

Magnetic Flux and Gauss' Law for Magnetism

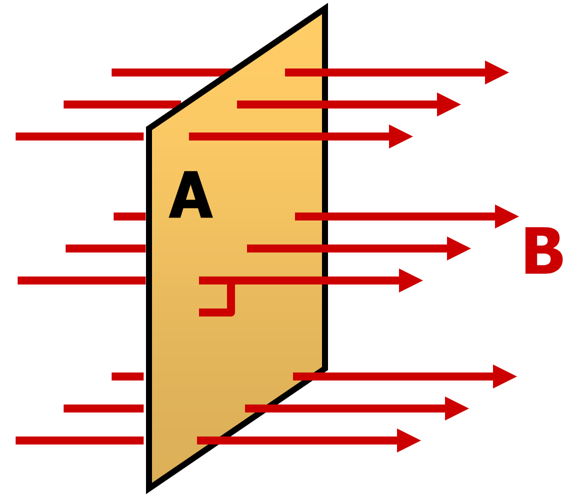
Magnetic Flux

We have used magnetic field lines to visualize magnetic fields and indicate their strength.

We are now going to count the number of magnetic field lines passing through a surface, and use this count to determine the magnetic field.

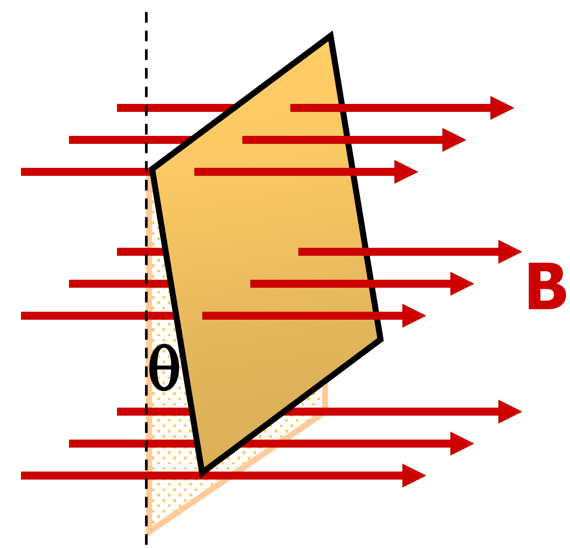


The magnetic flux passing through a surface is the number of magnetic field lines that pass through it.

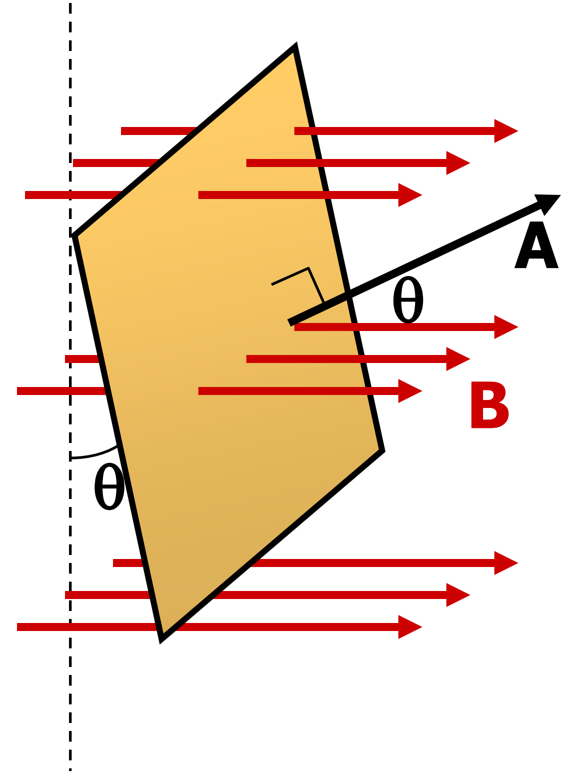


Because magnetic field lines are drawn arbitrarily, we quantify magnetic flux like this: $\Phi_M = BA$.

If the surface is tilted, fewer lines cut the surface.



We define $\vec{A}$ to be a vector having a magnitude equal to the area of the surface, in a direction normal to the surface.



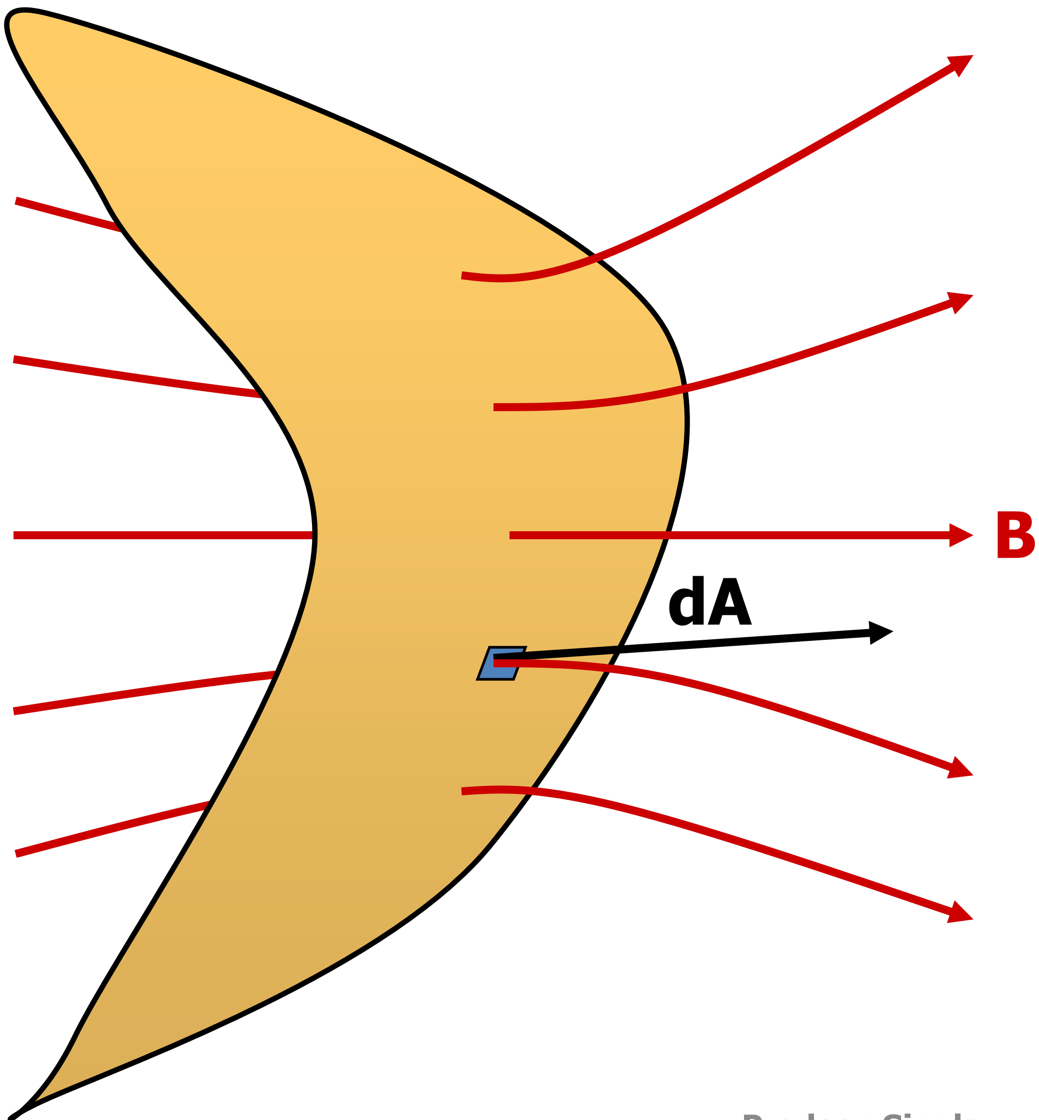
The "amount of surface" perpendicular to the magnetic field is $A\cos\theta$.

Because $\vec{A}$ is perpendicular to the surface, the amount of $\vec{A}$ parallel to the electric field is $A\cos\theta$.

$$A_{\parallel} = A\cos\theta \quad \text{so} \quad \Phi_M = BA_{\parallel} = BA\cos\theta.$$

Remember the dot product from Physics 23? $\Phi_M = \vec{B}\cdot\vec{A}$

If the magnetic field is not uniform, or the surface is not flat…



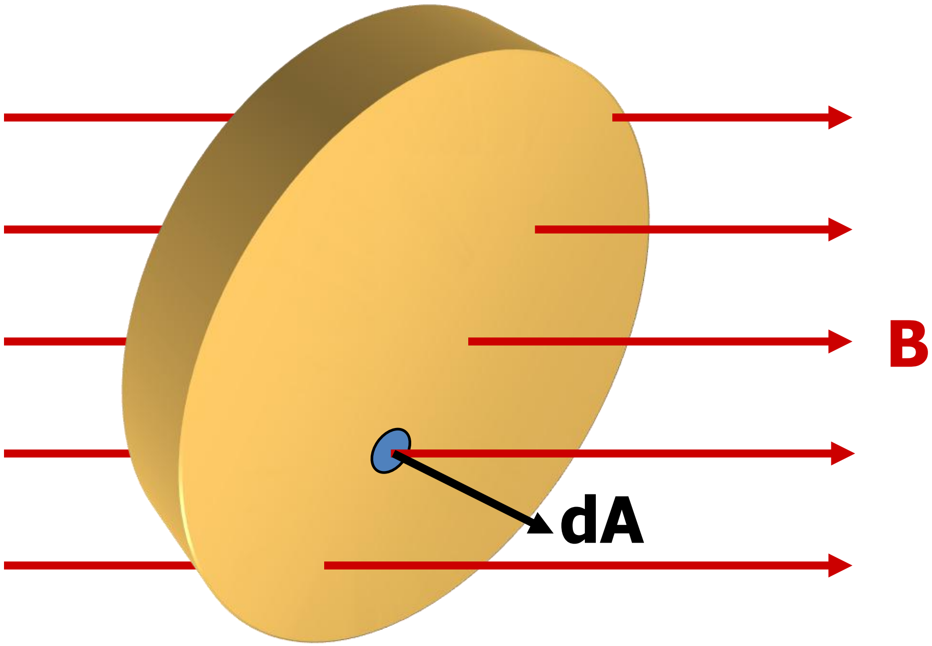
divide the surface into infinitesimal surface elements and add the flux through each…

$$\Phi_M = \lim_{\Delta A_i \to 0} \sum_i \vec{B}_i \cdot \Delta\vec{A}_i$$

$$\Phi_M = \int \vec{B} \cdot d\vec{A}$$

your starting equation sheet has. $\Phi_B = \int \vec{B} \cdot d\vec{A}$

If the surface is closed (completely encloses a volume)…

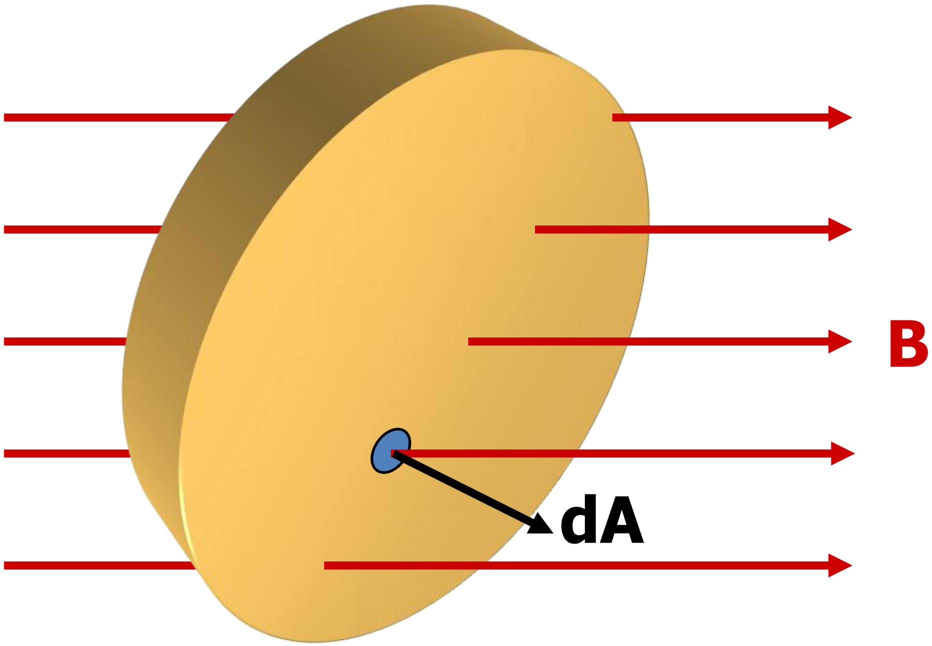


…we count lines going out as positive and lines going in as negative…

$$\Phi_M = \oiint \vec{B} \cdot d\vec{A}$$

a surface integral, therefore a double integral $\iint$

But there are no magnetic monopoles in nature (experimental fact). If there were more flux lines going out of than into the volume, there would be a magnetic monopole inside.



Therefore

$$\Phi_M = \oint \vec{B} \cdot d\vec{A} = 0$$

Gauss' Law for

Gauss' Law for magnetism is not very useful in this course. The concept of magnetic flux is extremely useful, and will be used later!

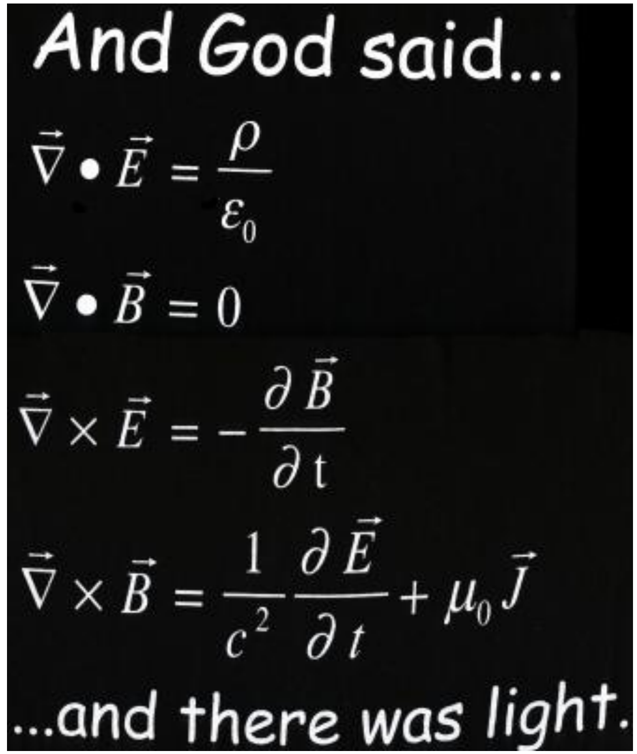
You have now learned Gauss's Law for both electricity and magnetism.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

These equations can also be written in differential form:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

The Missouri S&T Society of Physics Student T-Shirt!



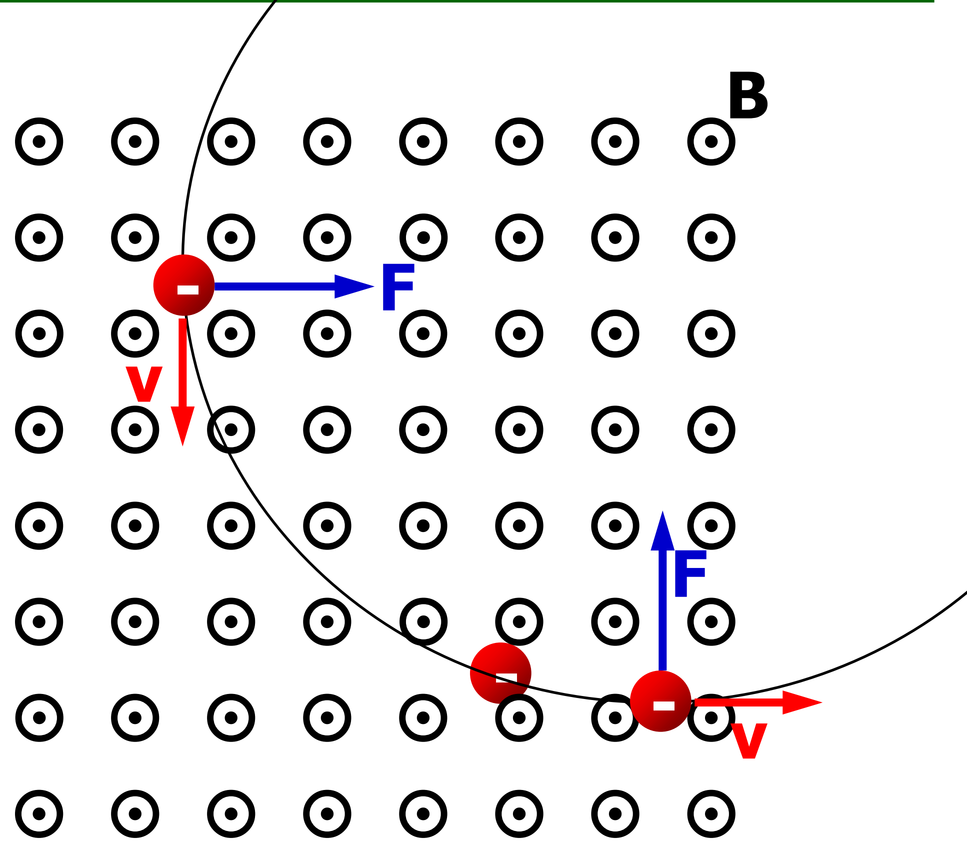
Motion of a charged particle in a uniform magnetic field

Example: an electron travels at 2×10^7 m/s in a plane perpendicular to a 0.01 T magnetic field. Describe its

Example: an electron travels at 2×10^7 m/s in a plane perpendicular to a 0.01 T magnetic field. Describe its

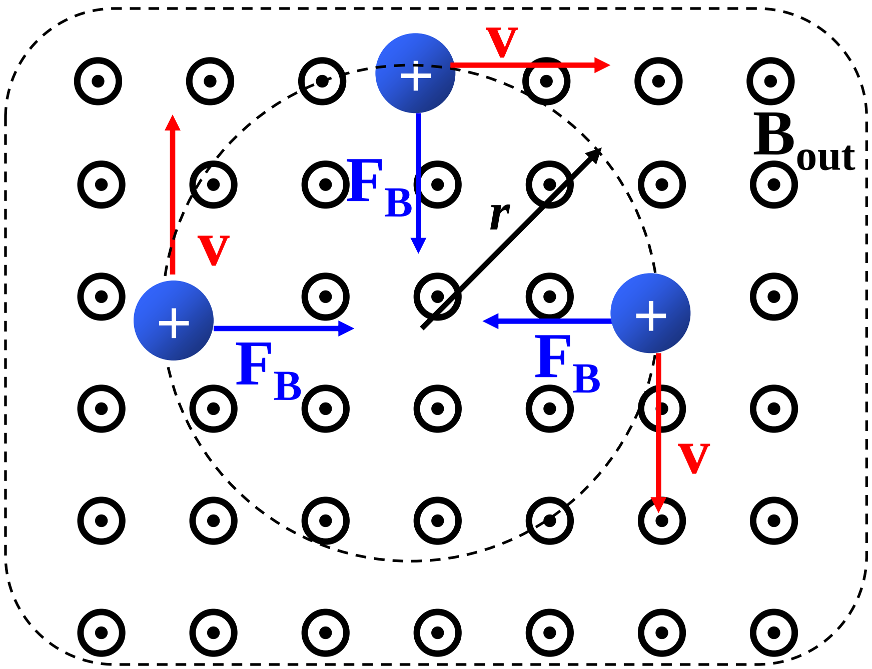
The force on the electron (remember, its charge is -) is always perpendicular to the velocity. If $\vec{v}$ and $\vec{B}$ are constant, then $\vec{F}$ remains constant (in magnitude).

The above paragraph is a description of uniform circular motion.



The electron will move in a circular path with an acceleration equal to v^2/r, where r is the radius of the circle.

Motion of a proton in a uniform magnetic field



Thanks to Dr. Waddill for the use of the picture and following examples.

The force is always in the radial direction and has a magnitude qvB. For circular motion, a = v²/r so

$$F = |q|vB = \frac{mv^2}{r}$$

$$v = \frac{|q|rB}{m} \qquad r = \frac{mv}{|q|B}$$

The period T is

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{|q|B}$$

The rotational frequency f is called the cyclotron frequency

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m}$$

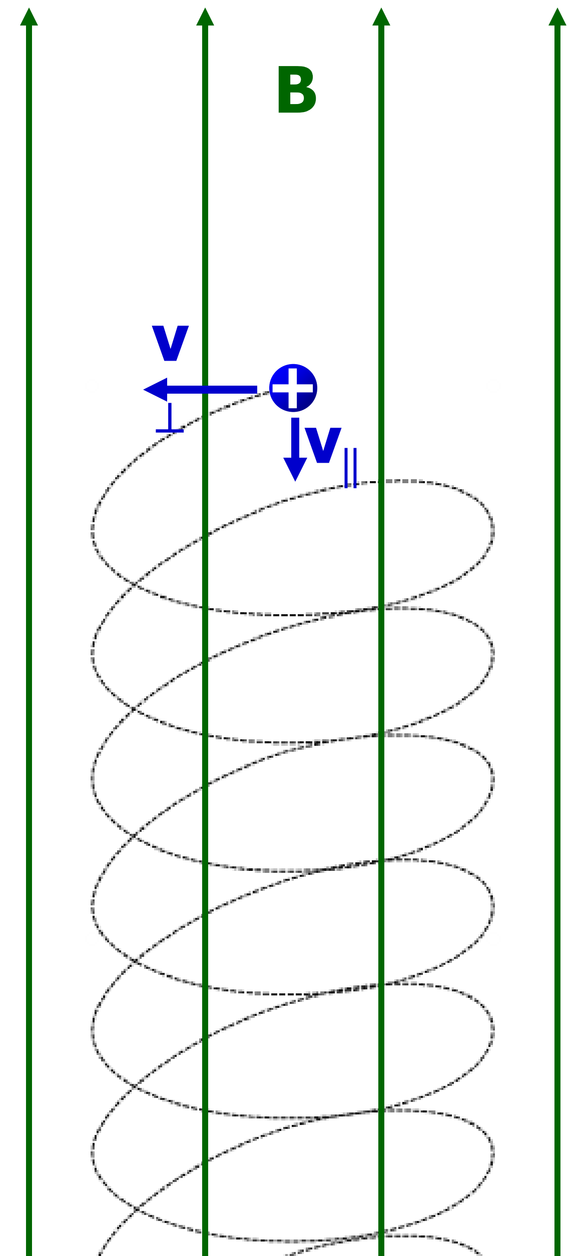
Remember: you can do the directions "by hand" and calculate using magnitudes only.

Helical motion in a uniform magnetic field

If $\vec{v}$ and $\vec{B}$ are perpendicular, a charged particle travels in a circular path. v remains constant but the direction of $\vec{v}$ constantly changes.

If $\vec{v}$ has a component parallel to $\vec{B}$, then $v_{\parallel}$ remains constant, and the charged particle moves in a helical path.

There won't be any test problems on helical motion.



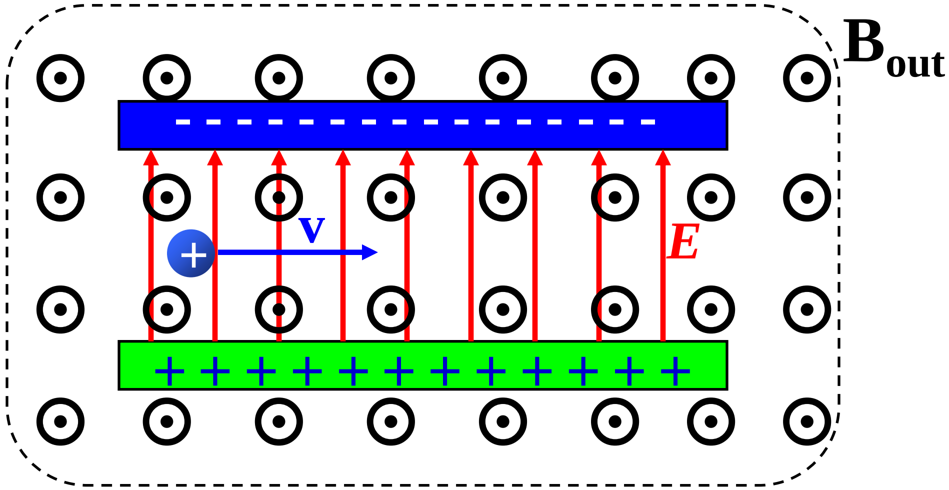
Lorentz Force Law

If both electric and magnetic fields are present, $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right).$

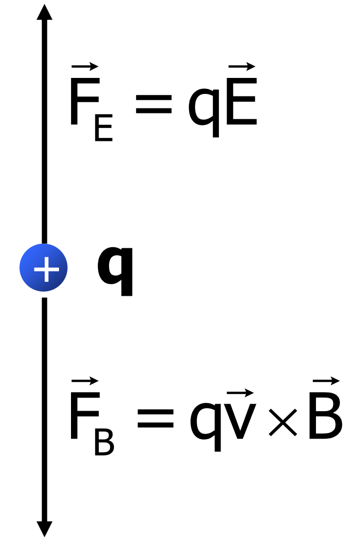
Applications

Velocity selector
Mass spectrometer

Velocity Selector



Thanks to Dr. Waddill for the use of the picture.



When the electric and magnetic forces balance then the charge will pass straight through. This occurs when $F_E = F_B$ or

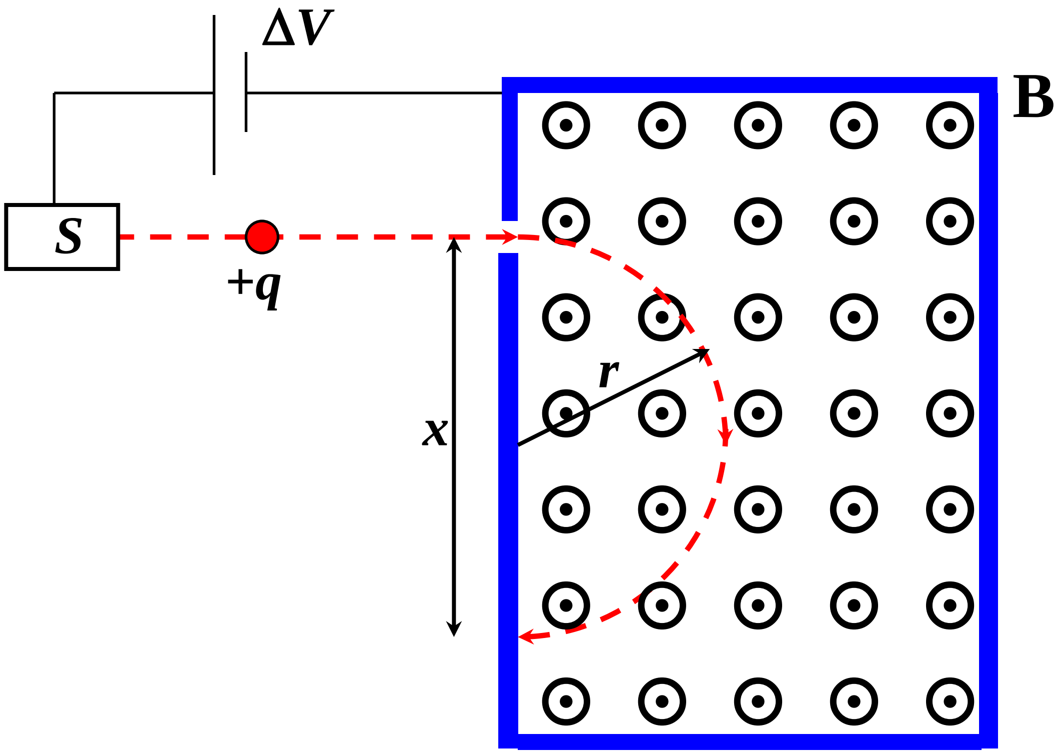
$$qE = qvB \quad \text{or} \quad v = \frac{E}{B}$$

Mass Spectrometers

Mass spectrometers separate charges of different mass.

When ions of fixed energy enter a region of constant magnetic field, they follow a circular path.

The radius of the path depends on the mass/charge ratio and speed of the ion, and the magnitude of the magnetic field.



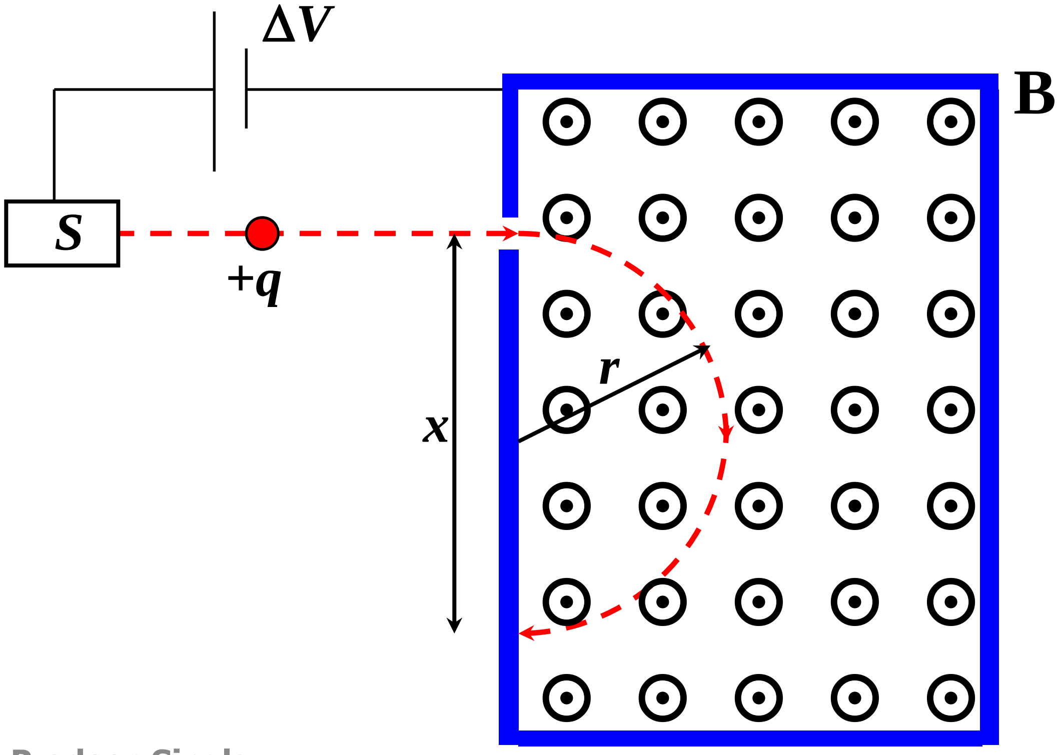
Thanks to Dr. Waddill for the use of the picture.

Example: Ions from source S enter a region of constant magnetic field B that is perpendicular to the ions path. The ions follow a semicircle and strike the detector plate at x = 1.7558 m from the point where they entered the field. If the ions have a charge of 1.6022 x 10^{-19} C, the magnetic field has a magnitude of B = 80.0 mT, and the accelerating potential is ΔV = 1000.0 V, what is the mass of the ion?

Radius of ion path:

$$x = 2r \quad \text{and} \quad r = \frac{mv}{qB}$$

Unknowns are m and v.



Pradeep Singla

Conservation of energy gives speed of ion. The ions leave the source with approximately zero kinetic energy

$$K_i + U_i = K_f + U_f$$

$$K_f = \cancel{K_i}^{\,0} + (U_i - U_f)$$

$$K_f = -(U_f - U_i) = -q(V_f - V_i) = -q\Delta V$$

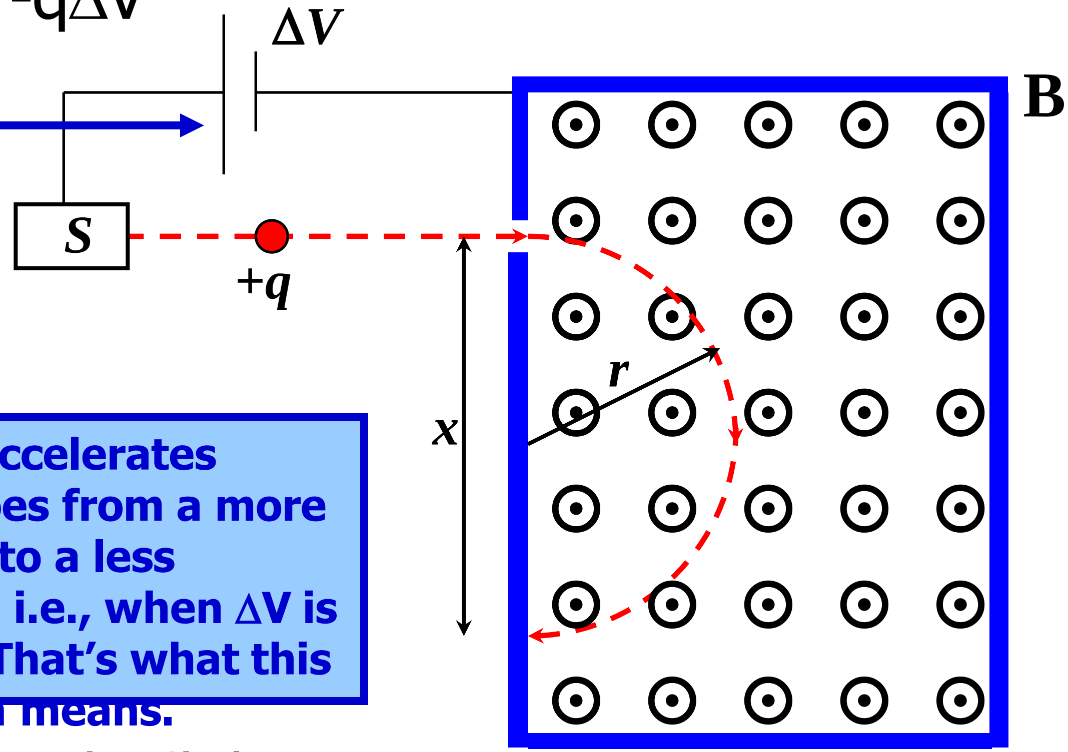
$$K_f = -q\Delta V$$

$$\frac{1}{2}mv^2 = -q\Delta V$$

$$v = \sqrt{\frac{-2q\Delta V}{m}}$$

Caution! V is potential, v (lowercase) is speed.

A proton accelerates when it goes from a more positive V to a less positive V; i.e., when ΔV is negative. That's what this minus sign means.



Pradeep Singla

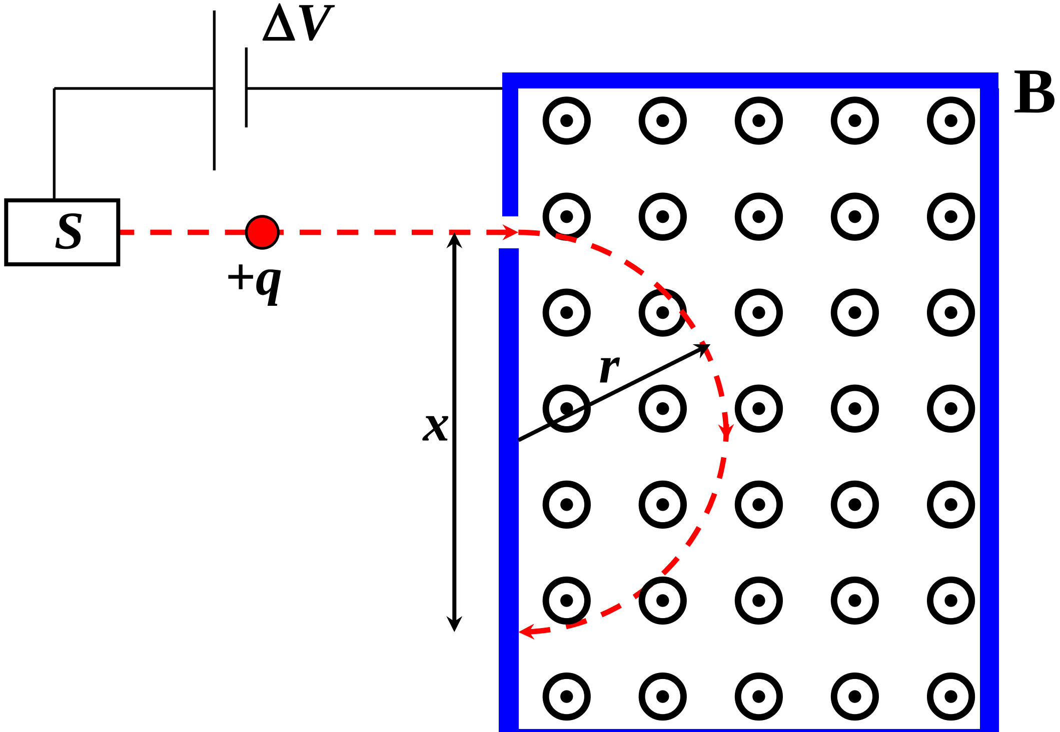
$$v = \sqrt{\frac{-2q\Delta V}{m}}$$

$$x = 2r = \frac{2mv}{qB}$$

$$x = \frac{2m}{qB}\sqrt{\frac{-2q\Delta V}{m}}$$

$$x = \frac{2}{B}\sqrt{\frac{-2m\Delta V}{q}}$$

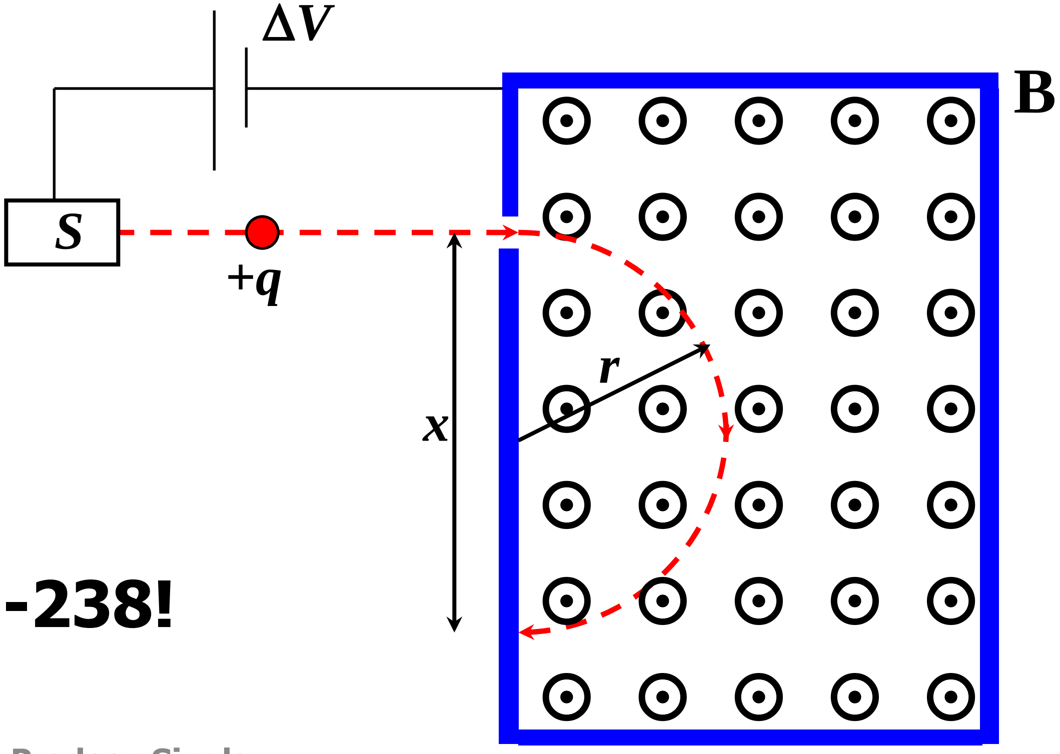
$$m = -\frac{B^2x^2q}{8\Delta V}$$



$$m = -\frac{B^2 x^2 q}{8\Delta V}$$

$$m = -\frac{(0.08\ \text{T})^2 (1.7558\ \text{m})^2 (1.6022\times10^{-19}\text{C})}{8(-1000\ \text{V})}$$

$$m = 3.9515\times10^{-25}\text{kg}$$



1 atomic mass unit (amu) equals 1.66x10^{-27} kg, so

$m = 238.04\ \text{u}$ **uranium-238!**

Reminder: signs

$$\vec{F} = q\vec{v} \times \vec{B}$$

Include the sign on q, properly account for the directions of any two of the vectors, and the direction of the third vector is calculated "automatically."

$$F = |q| vB \sin\theta = |q| v_{\perp} B = |q| vB_{\perp}$$

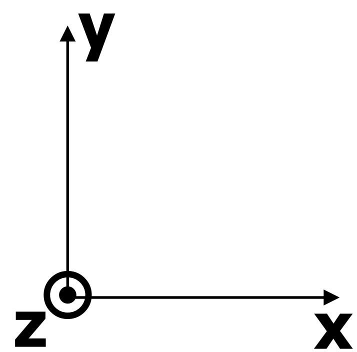
If you determine the direction "by hand," use the magnitude of the charge.

Everything in this equation is a magnitude. The sign of r had better be +!

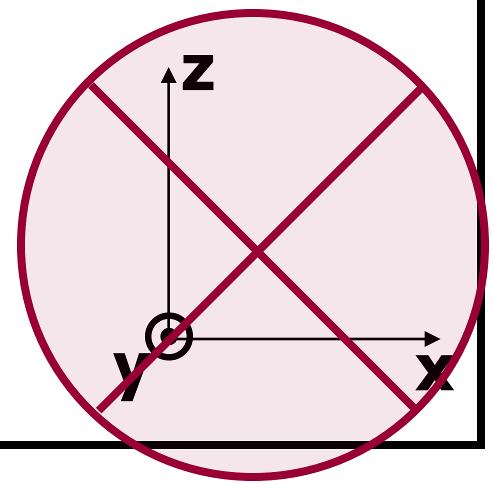
$$r = \frac{mv}{|q|B}$$

Reminder: left- and right-hand axes

This is a right-handed coordinate system:

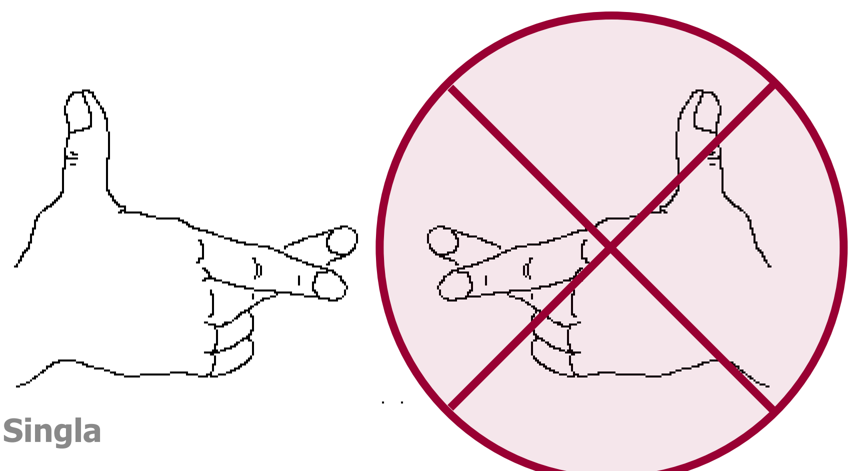


This is not:

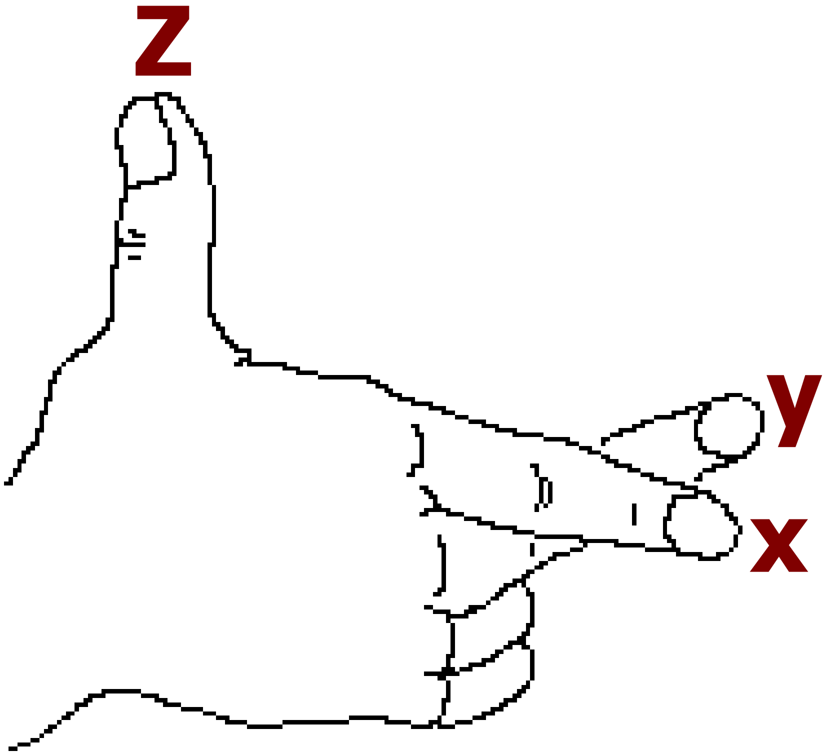


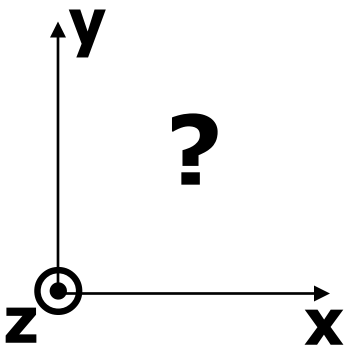
For the magnetism part of physics 24, you MUST use right-hand axes.

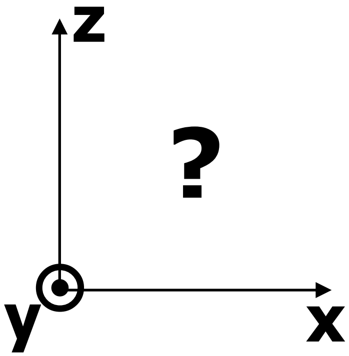
And you'd better use your right hand when applying the right-hand rule!



Handy way to "see" if you have drawn right-hand axes:







Magnetic and Electric Forces

The electric force acts in the direction of the electric field.

$$\vec{F}_E = q\vec{E}$$

The electric force is nonzero even if v=0.

The magnetic force acts perpendicular to the magnetic field.

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

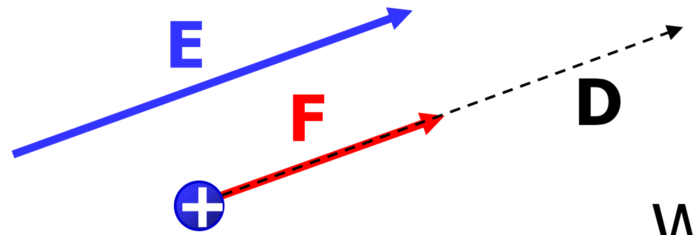
The magnetic force is zero if v=0.

$$\vec{F}_B\left(v=0\right) = q\left(0\right) \times \vec{B} = 0$$

Magnetic and Electric Forces

The electric force does work in displacing a charged particle.

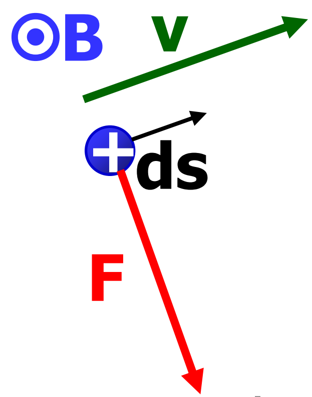
$$\vec{F}_E = q\vec{E}$$



$$W_F = \vec{F}\cdot\vec{D} = FD = qED$$

The magnetic force does no work in displacing a charged particle!

$$\vec{F}_B = q\vec{v}\times\vec{B}$$



$$W_F = \vec{F}\cdot d\vec{s} = 0$$

Amazing!

Pradeep Singla

Magnetic Forces on Currents

So far, I've lectured about magnetic forces on moving charged particles.

$$\vec{F} = q\vec{v} \times \vec{B}$$

Actually, magnetic forces were observed on current-carrying wires long before we discovered what the fundamental charged particles are.

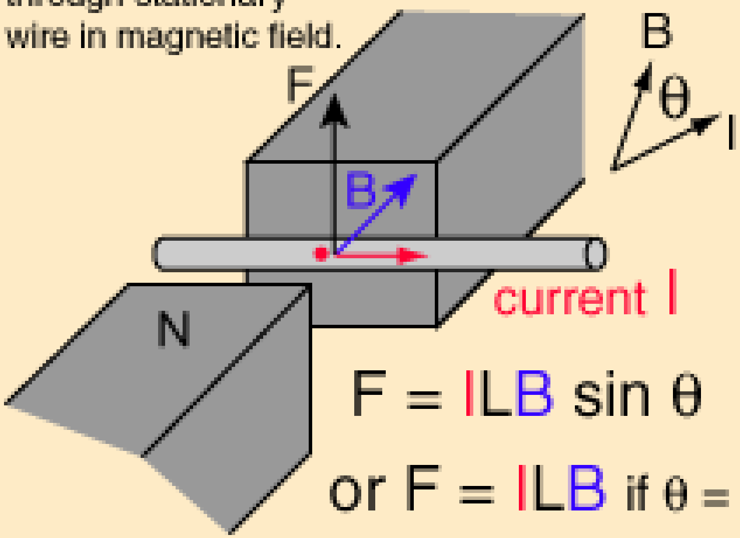
Experiment, followed by theoretical understanding, gives

$$\boxed{\vec{F} = I\vec{L} \times \vec{B}.}$$

For reading clarity, I'll use L instead of the ℓ your text uses.

If you know about charged particles, you can derive this from the equation for the force on a moving charged particle. It is valid for a straight wire in a uniform magnetic field.

Positive charge moving through stationary wire in magnetic field.



$$F = ILB \sin\theta$$

or $F = ILB$ if $\theta = 90^\circ$

This relationship arises from the basic magnetic force:

$$F = qvB \sin\theta$$

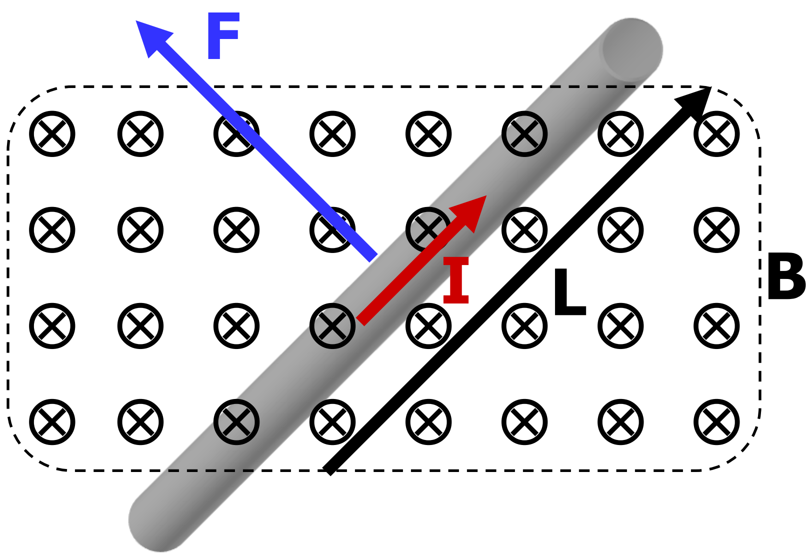
which for a charge q traveling length L in a wire can be written

$$F = q\frac{L}{t} B \sin\theta$$

$$F = \frac{q}{t} LB \sin\theta$$

$$F = ILB \sin\theta$$

$$|F| = ILB \sin\theta \Rightarrow |F|_{max} = ILB$$



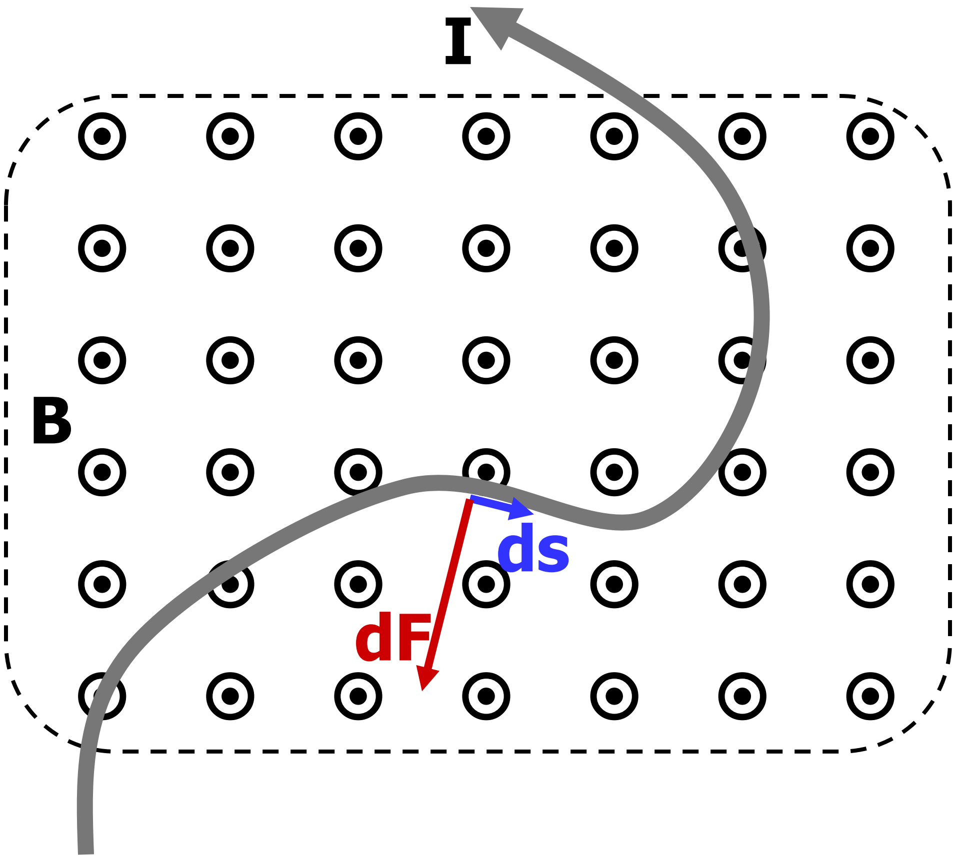
$$\vec{F} = I\vec{L} \times \vec{B}$$

Valid for straight wire, length L inside region of magnetic field,
constant magnetic field, constant current I, direction of $\vec{L}$ is direction of conventional current I.

You could apply this equation to a beam of charged particles moving through space, even if the charged particles are not confined to a wire.

Pradeep Singla

What if the wire is not straight?



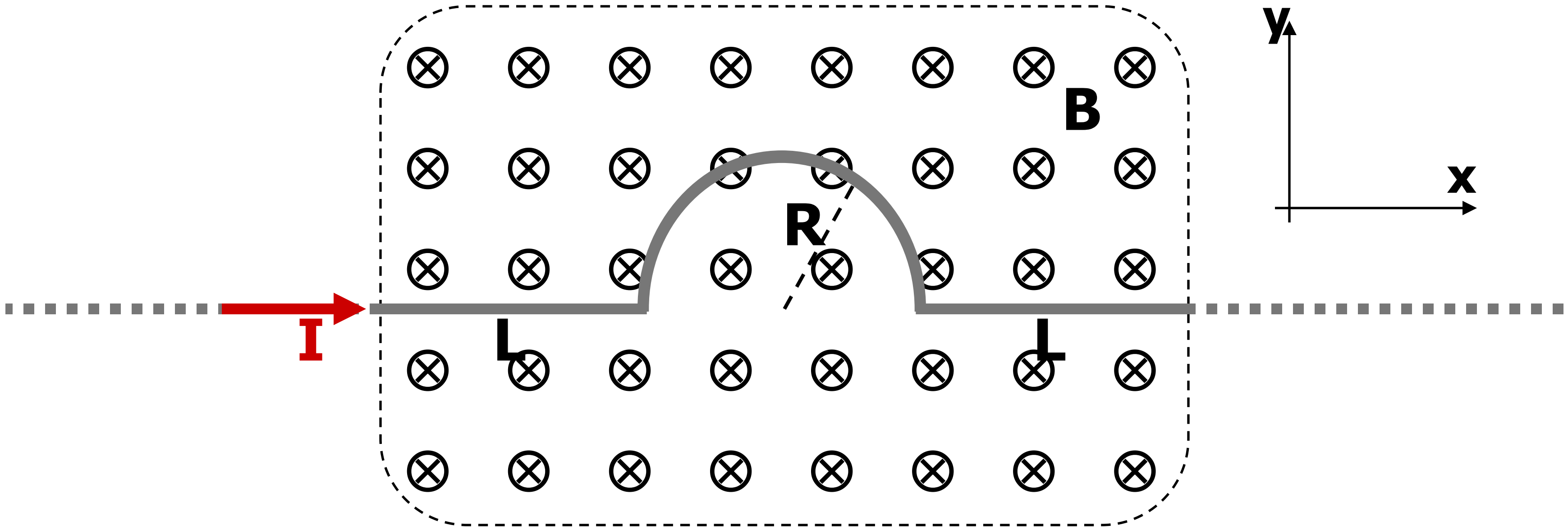
$$d\vec{F} = I\, d\vec{s} \times \vec{B}$$

$$\vec{F} = \int d\vec{F}$$

$$\vec{F} = I \int \left(d\vec{s} \times \vec{B} \right)$$

Integrate over the part of the wire that is in the magnetic field region.

Example: a wire carrying current I consists of a semicircle of radius R and two horizontal straight portions each of length L. It is in a region of constant magnetic field as shown. What is the net magnetic



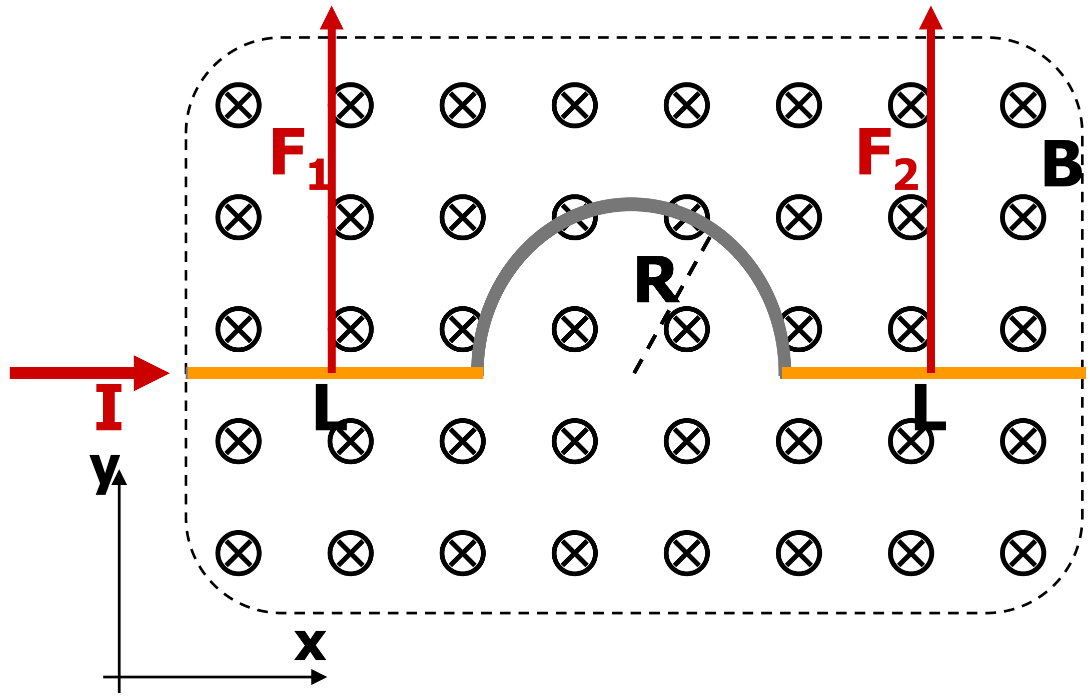
There is no magnetic force on the portions of the wire outside the magnetic field region.

First look at the two straight sections.

$$\vec{F} = I\vec{L} \times \vec{B}$$

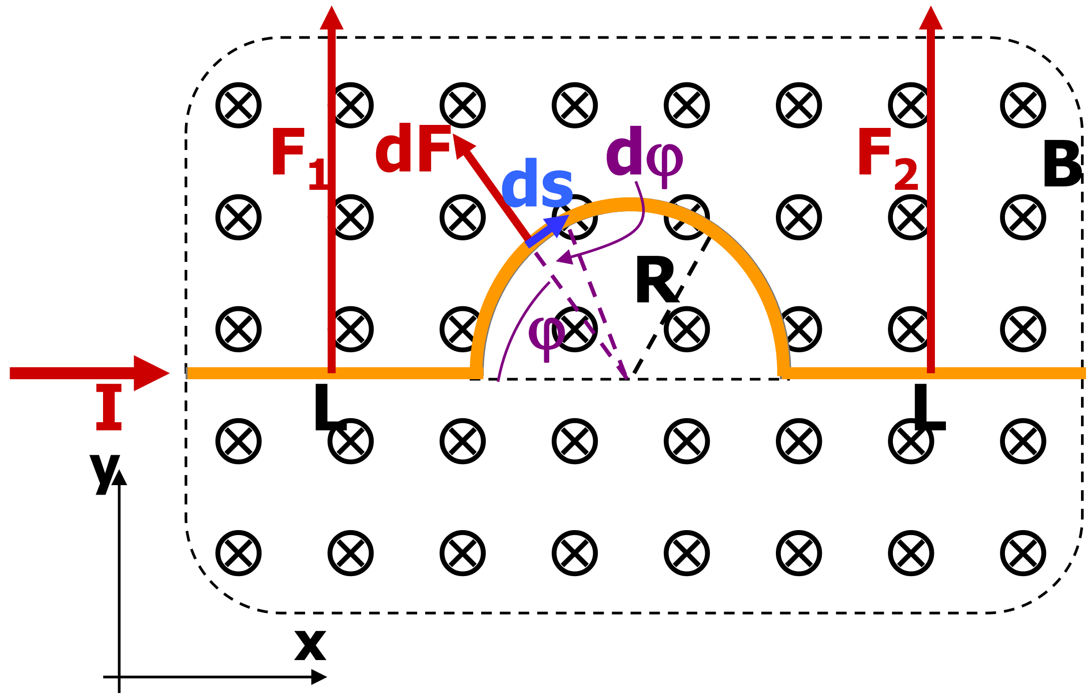
$\vec{\mathbf{L}} \perp \vec{\mathbf{B}}$**, so**

$$F_1 = F_2 = ILB$$



Next look at the semicircular section.

Calculate the incremental force $\vec{dF}$ on an incremental $\vec{ds}$ of current-carrying wire.



ds subtends the angle from φ to φ+dφ.

The incremental force is $d\vec{F} = I\, d\vec{s} \times \vec{B}$.

$\vec{ds} \perp \vec{B}$**, so** $dF = I\, ds\, B$.

Arc length $ds = R\, d\varphi$.

Finally, $dF = I\, R\, d\varphi\, B$.

Calculate the y-component of F.

$$dF_y = I\,R\,d\varphi\,B\,\sin\varphi$$

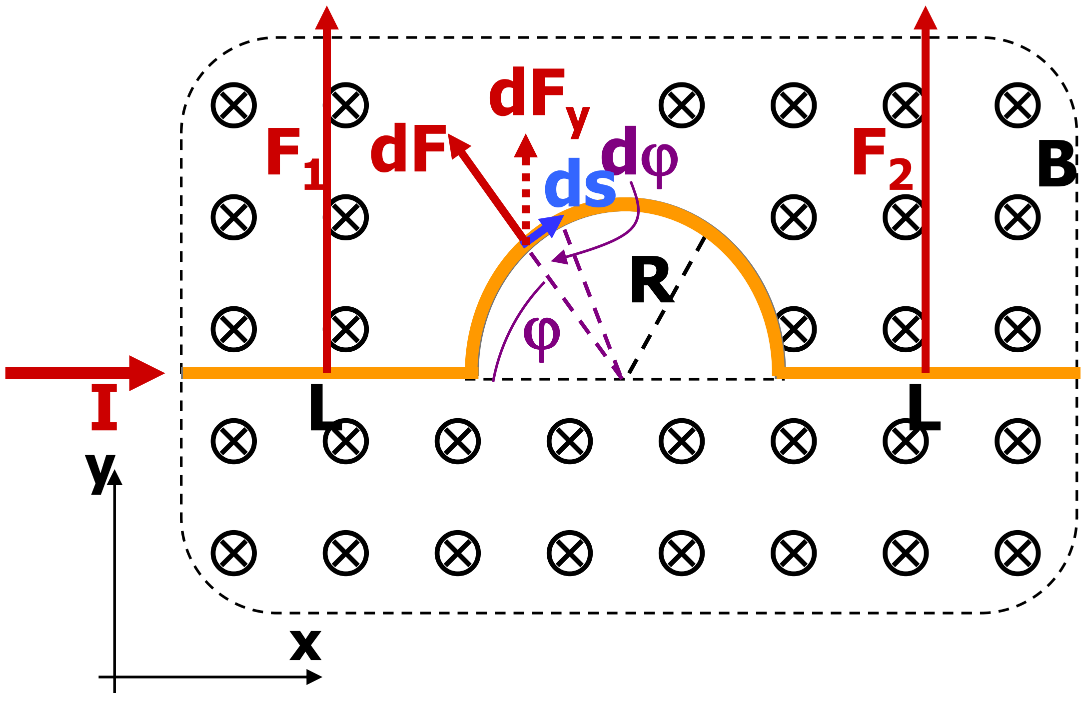
$$F_y = \int_0^{\pi} dF_y$$

$$F_y = \int_0^{\pi} I\,R\,d\varphi\,B\,\sin\varphi$$

$$F_y = I\,R\,B\int_0^{\pi} \sin\varphi\,d\varphi$$

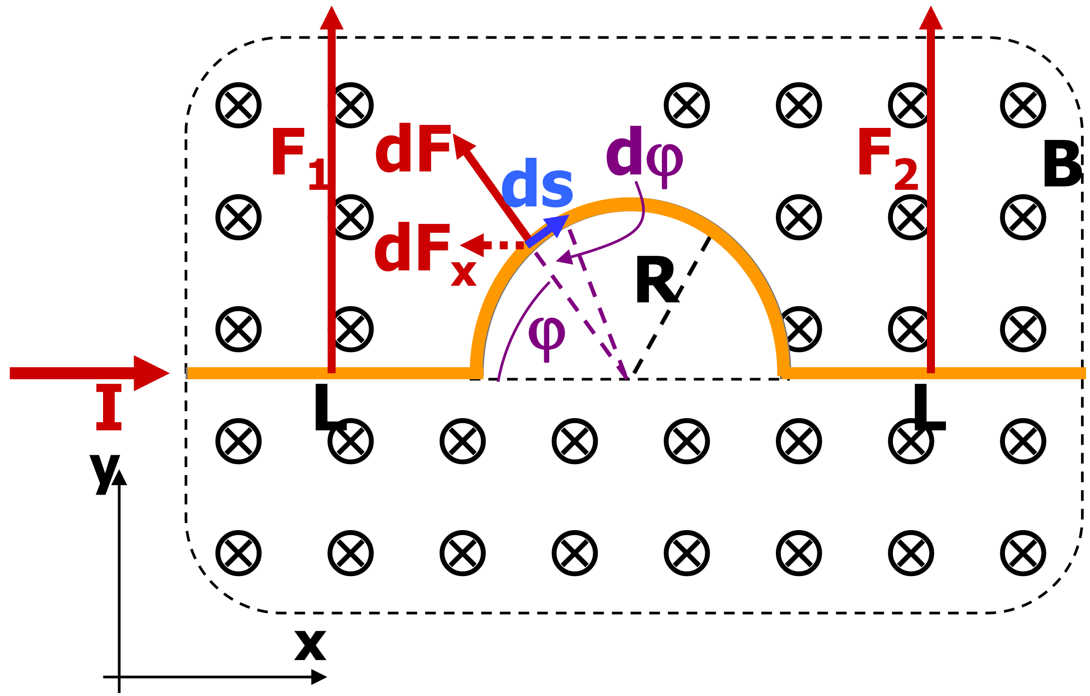
$$F_y = \left(-I\,R\,B\cos\varphi\right)\Big|_0^{\pi}$$

$$F_y = 2\,I\,R\,B$$



Pradeep Singla

Does symmetry give you F_x immediately?
Or, you can calculate the x component of F.



$$dF_x = -I\,R\,d\varphi\,B\cos\varphi$$

$$F_x = -\int_0^{\pi} I\,R\,d\varphi\,B\cos\varphi$$

$$F_x = -I\,R\,B\int_0^{\pi}\cos\varphi\,d\varphi$$

$$F_x = -\left(I\,R\,B\sin\varphi\right)\Big|_0^{\pi}$$

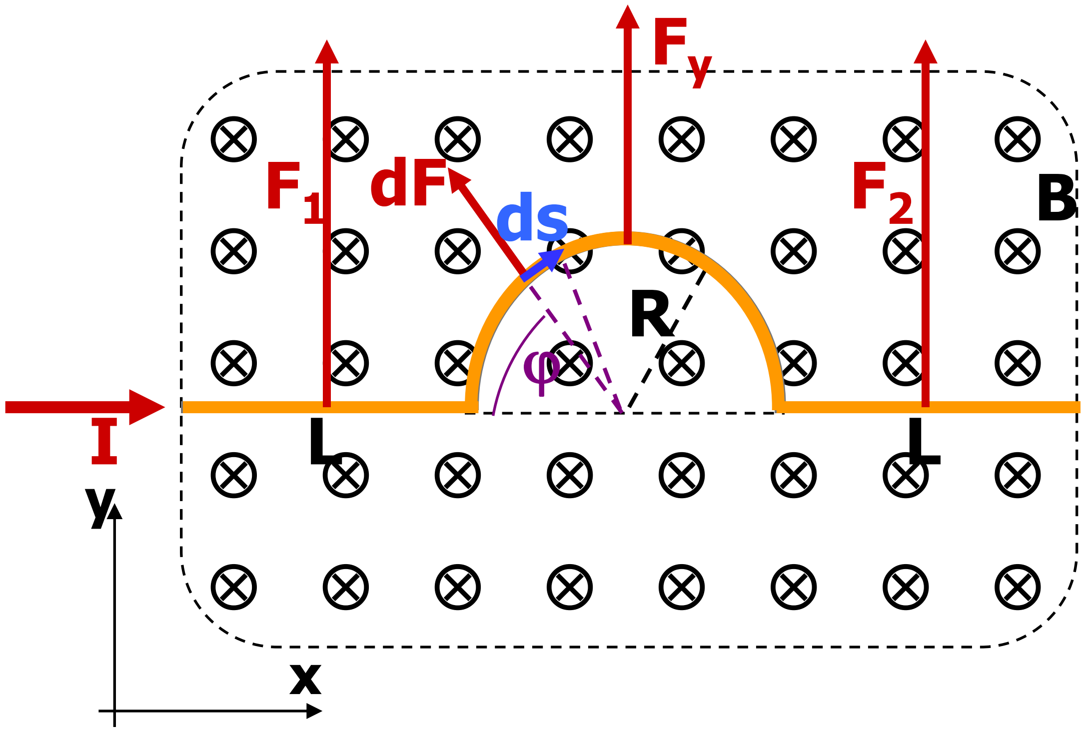
$$F_x = 0$$

Total force:

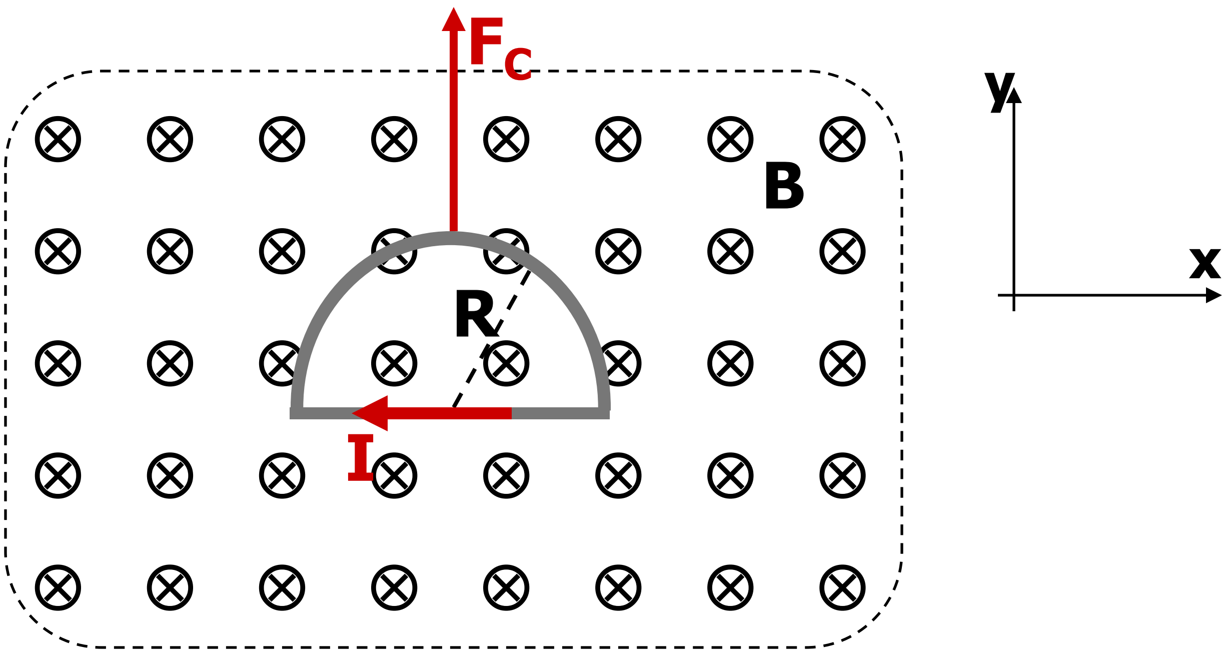
$$F = F_1 + F_2 + F_y$$

$$F = ILB + ILB + 2IRB$$

$$F = 2IB(L + R)$$



Example: a semicircular closed loop of radius R carries current I. It is in a region of constant magnetic field as shown. What is the net magnetic force on the loop of



We calculated the force on the semicircular part in the previous example (current is flowing in the same direction there as before).

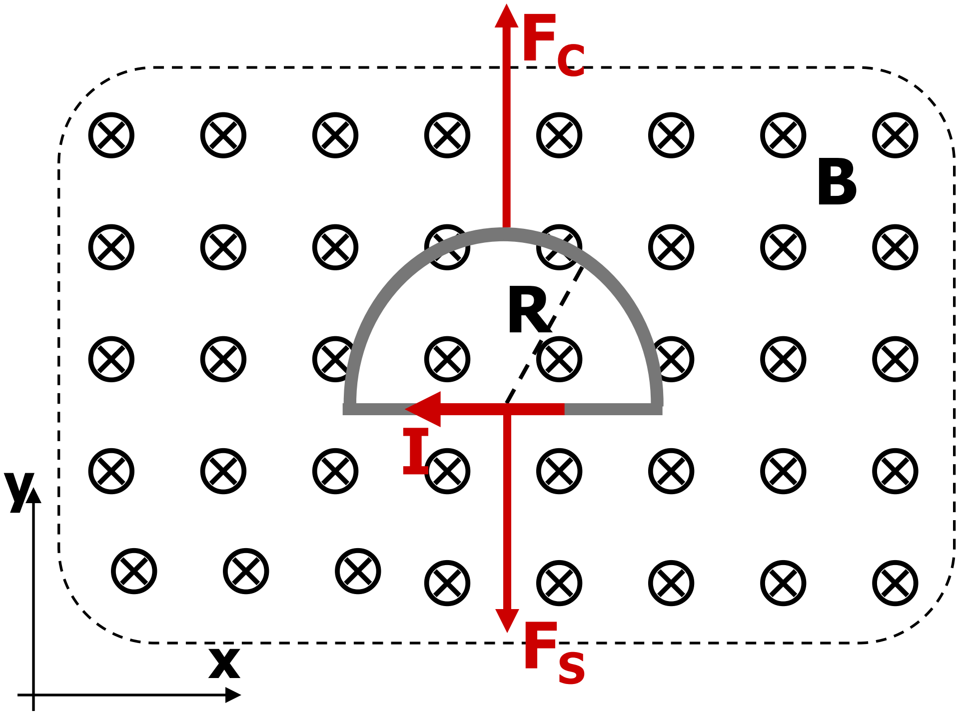
$$F_C = 2\,I\,R\,B$$

Next look at the straight section.

$$\vec{F}_S = I\vec{L} \times \vec{B}$$

$\vec{L} \perp \vec{B}$, and L=2R so

$$F_S = 2IRB$$



$\vec{F}_s$ is directed in the –y direction (right hand rule).

$$\vec{F}_{net} = \vec{F}_S + \vec{F}_c = -2IRB\,\hat{j} + 2IRB\,\hat{j} = 0$$

The net force on the closed loop is zero!

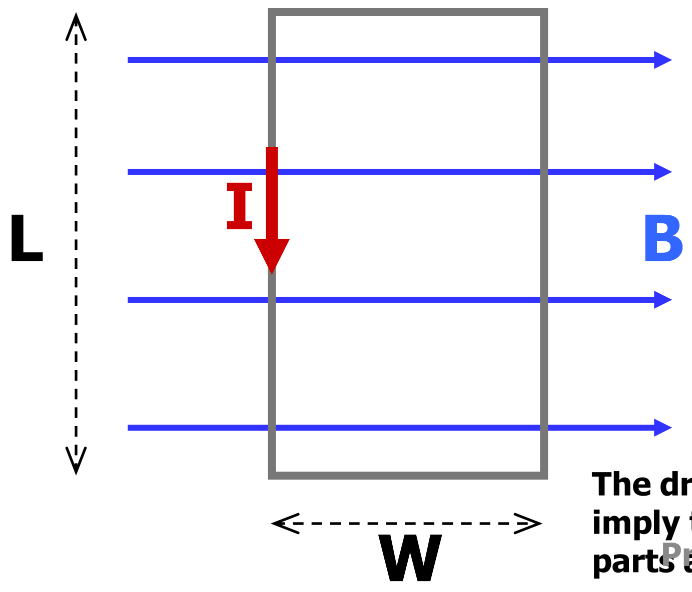
This is true in general for closed loops in a uniform magnetic field.

Magnetic Forces and Torques on Current Loops

We showed (not in general, but illustrated the technique) that the net force on a current loop in a uniform magnetic field is zero.

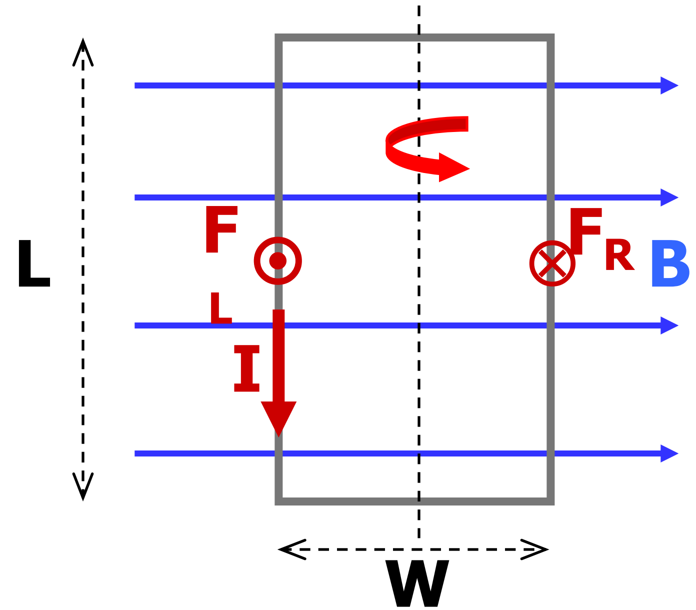
No net force means no motion.NOT.

Example: a rectangular current loop of area A is placed in a uniform magnetic field. Calculate the torque on the l



Let the loop carry a counterclockwise current I and have length L and width W.

The drawing is not meant to imply that the top and bottom parts are outside the magnetic field region.

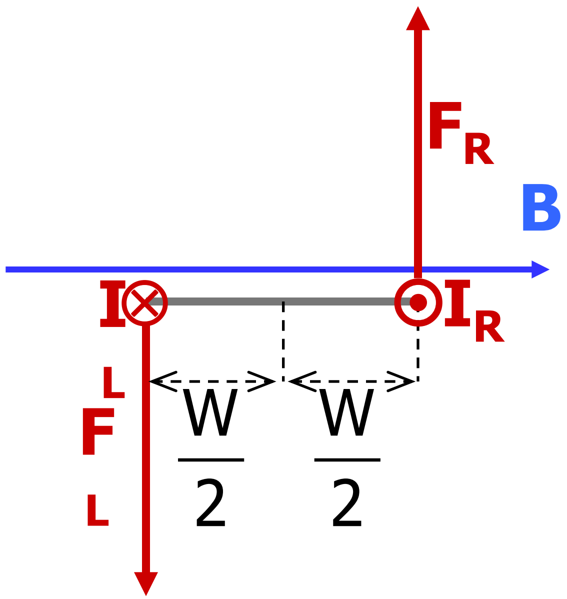


There is no force on the "horizontal" segments because the current and magnetic field are in the same direction.

The vertical segment on the left "feels" a force "out of the page."
The vertical segment on the right "feels" a force "into the page."
The two forces have the same magnitude: $F_L = F_R = I\,L\,B$.
Because $\vec{F}_L$ and $\vec{F}_R$ are in opposite directions, there is no net force on the current loop, but there is a net torque.

Top view of current loop, looking "down," at the instant the magnetic field is parallel to the plane of the loop.

In general, torque is $\vec{\tau} = \vec{r} \times \vec{F}$.



$$\tau_R = \frac{W}{2} F_R = \frac{1}{2} WILB$$

$$\tau_L = \frac{W}{2} F_L = \frac{1}{2} WILB$$

$$\tau_{net} = \tau_R + \tau_L = WILB = IAB$$

area of loop = WL

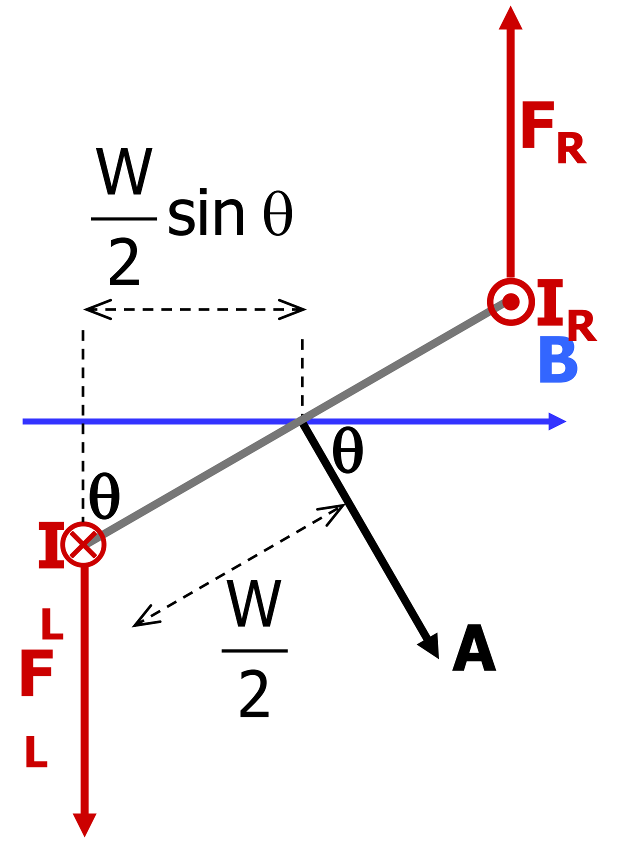
Pradeep Singla

When the magnetic field is not parallel to the plane of the loop...

$$\tau_R = \frac{W}{2} F_R \sin\theta = \frac{1}{2} WILB \sin\theta$$

$$\tau_L = \frac{W}{2} F_L \sin\theta = \frac{1}{2} WILB \sin\theta$$

$$\tau_{net} = \tau_R + \tau_L = WILB \sin\theta = IAB \sin\theta$$



Define $\vec{A}$ to be a vector whose magnitude is the area of the loop and whose direction is given by the right hand rule (cross $\vec{A}$ into $\vec{B}$ to get $\vec{\tau}$). Then

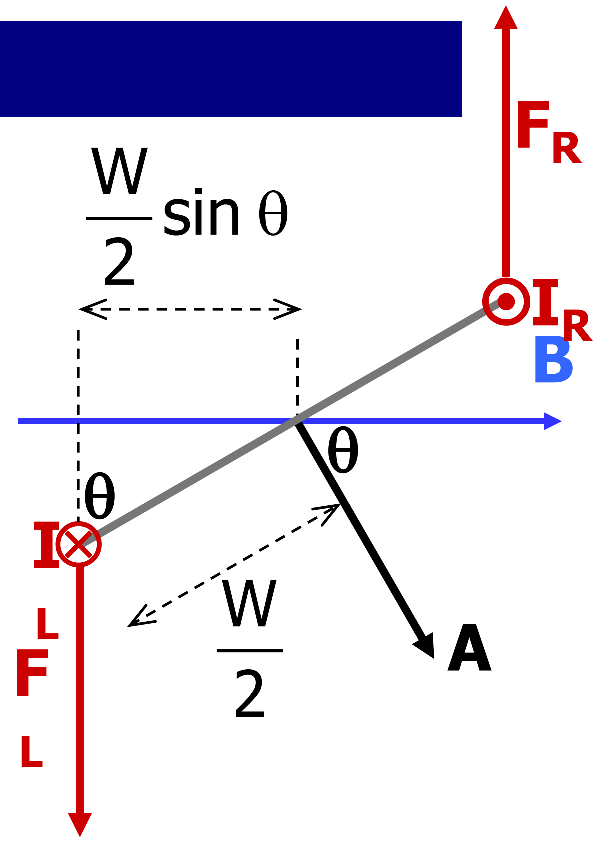
$$\vec{\tau} = I\vec{A} \times \vec{B}.$$

Magnetic Moment of a Current Loop

$$\vec{\tau} = I\vec{A} \times \vec{B}$$

Alternative way to get direction of $\vec{A}$: curl your fingers (right hand) around the loop in the direction of the current; thumb points in direction of $\vec{A}$.

$I\vec{A}$ is defined to be the magnetic moment of the current loop.



$$\vec{\mu} = I\vec{A}$$

$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$

Your starting equation sheet has:

$$\boxed{\vec{\mu} = N\,I\,\vec{A} \qquad (N = 1 \text{ for a single loop})}$$

Energy of a magnetic dipole in a magnetic field

You don't realize it yet, but we have been talking about magnetic dipoles for the last 5 slides.

A current loop, or any other body that experiences a magnetic torque as given above, is called a magnetic dipole.

Energy of a magnetic dipole?

You already know this:

Electric Dipole

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

Today:

Magnetic Dipole

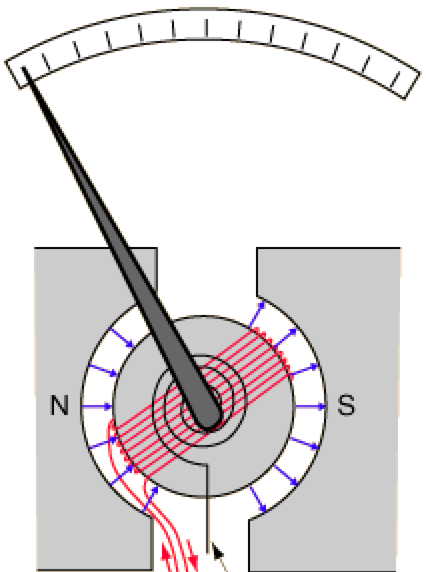
$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$U = -\vec{\mu} \cdot \vec{B}$$

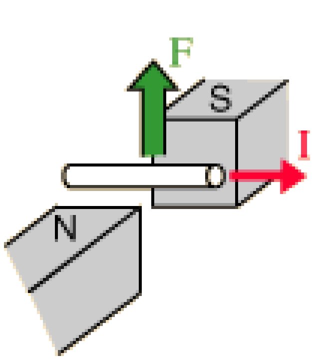
The Galvanometer

Now you can understand how a galvanometer works…

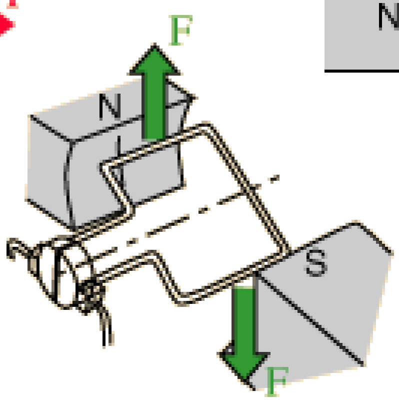
When a current is passed through a coil connected to a needle, the coil experiences a torque and deflects. See the link below for more details.



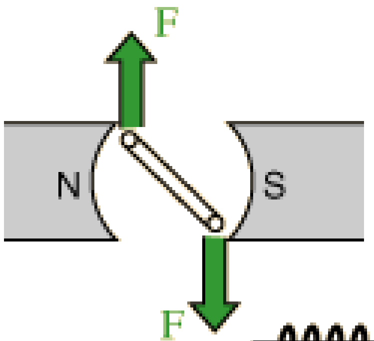
Electric Motors



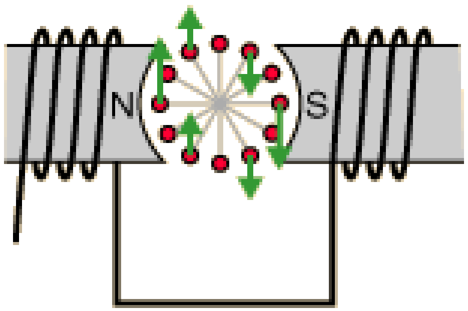
An electric current in a magnetic field will experience a force.



If the current-carrying wire is bent into a loop, then the two sides of the loop which are at right angles to the magnetic field will experience forces in opposite directions.



The pair of forces creates a turning influence or torque to rotate the coil.



Practical motors have several loops on an armature to provide a more uniform torque and the magnetic field is produced by an electromagnet arrangement called the field coils.

Hyperphysics has nice interactive graphics showing how dc and ac motors work.